OPTIMUM USE OF WATER RESOURCES

## OPTIMUM USE OF WATER RESOURCES

N. A. DE RIDDER
A. EREZ


Dr. N. A. de Ridder
Geo-hydrologist ILR] Wageningen, The Nethertands
Dr. A. Erez
Economist UNDP, Port of Spain, Trinidad \& Tobago

## Preface

Nowadays, problems of water resources operation, design, and planning are often solved by a systems approach. This was not so some 10 years ago when we, the authors of this publication, joined the FAO-UNDP Project: Integrated Planning of Irrigated Agriculture in the Varamin Plain, Iran. Our task was to draw up plans for the optimum supply of irrigation water.

During our work, we became more and more interested in the systems approach, as we became aware of the complicated economic, agronomic, and hydrologic problems involved. A storage dam was under construction in the main river, but as it controlled only a portion of the river's catchment, we were facing the problem of a stochastic supplý of surface water. Surface water was a limiting and variable resource, and much more land than could be irrigated was available. The solution therefore had to be sotight in the conjunctive use of the surface water resources and those of the groundwater basin. To help us with our problem, we developed a computerized groundwater model of the basin for use as a simulation tool.

Although the number of possible plans for the joint use of the two water resources is infinite, only a few plans are physically feasible. A primary objective in such plans is the continued use of the groundwater basin into che indefinite future. The mere development of $i$ ts resources would not, in itself, solve the region's irrigation supply problem; they must be properly managed as well. The groundwater basin must be operated at a safe-yield constraint level to prevent the inflow of saline groundwater from adjacent areas. That constraint could be released in water-deficient years to maintain the optimum area under irrigation, but any mining of the groundwater must be compensated for by artificial recharge with the excess river flow in spring.

Another problen we were facing was the economy of the operation; irrigation water must be supplied at reasonable cost. The costs of providing surface water or groundwater diffex; they also differ as to the site in the Plain where water is to be supplied. The idea was therefore born to apply linear programming as a tool to determine optimum solutions of irrigation water supply. The groundwater model could then be used to test these solutions for the impact they might have on the water table.

During the project we were able to develop the methodology for this approach, to develop, verify, and test the two models, and to use them for operational studies. Early in 1970 the project was terminated and we both returned to our home countries. Because each of us resumed our normal duties there, the issue of this publication has been much delayed. In 1974 the studies were recommenced at the International Institute for Land Reclamation and Improvenent (ILRI), Wageningen, The Netherlands. The linear programming model was slightly improved, all input data were verified, and a series of final plans were run on the models.

In their excellent handbook, "Water Resources Systems Engineering", HALL and DRACUP (1970) state: "Digital simulation has been extensively applied to water resources systems; however, it appears that further research is needed in the combination of optimization techniques and simulation in the analysis of these systems." In thi's publication we present the methodology and resules of our efforts to obtain this combination and we hope that our work may be a help and a stimulus to all others engaged in this field.

N.A.de Ridder

A.Erez

## Acknowledgements

The results presented in this publication could not have been realized without the help and cooperation of many individuals. We wish to acknowledge the advice and assistance of Mr.R.G.Thomas, Hydrogeologist, FAO, Rome, Mr.E.M.Weber and Mr.R.Y.D.Chun of the California Department of Water Resources and Consultants to the FAO-UNDP Project, all three of whom contributed considerably in developing and verifying the groundwater model. We are grateful to LBM, Teheran, for allowing us to use its computer facilities and for the most valuable assistance of Mr.Klint, IBM systems engineer.

In preparing the parameters and other input data for the models, we received the help of several of our colleagues and counterparts. In this respect we are much indebted to Dr.H.Diestel, FAO associate expert in bydrology, who generously satisfied our eagerness for surface water input data, and to Mr.A.S.Emadi, Iranian counterpart in hydrogeology, who contributed much in developing the groundwater model.

As far as the studies conducted in Wageningen are concerned, we are particularly grateful to Dr.G.C.Meyerman, agricultural economist of the University of Agriculture, Wageningen, for his great interest. in our work and his constructive criticism and suggestions for improving the linear programming model.

We also wish to thank Mr.J.B.H.M.van Gils of che Mathematical Department of the Institute for Land and Water Management Research, Wageningen, for adjusting the available computer programmes to the computer Eacilities in our building, and Mr.J. Boonstra, hydrologist of LLRI, for his careful review of the stochastic
river flow data used in the models and his effective cooperation in organizing, testing, adjusting, and evaluating the various test and final runs that were made on the computer models.

## Contents

Preface ..... v
Acknowledgements ..... vii
I.. Introduction ..... 1
2. Description of the Varamin Plain ..... 4
2.1 Location and extent ..... 4
2.2 Geomorphology ..... 4
2.3 Geology ..... 7
2.4 Land resources ..... 10
2.5 Climate ..... 13
2.6 Surface water resources ..... 13
2.6.1 Selecting a discharge distribution ..... 15
2.7 Groundwater resources ..... 21
2.7.1 Groundwater in storage and present recovery ..... 2 f
2.7.2 Groundwater quality ..... 26
2.8 Present system of irrigation water supply ..... 29
3. Digital groundwater basin model ..... - 31
3.1 Systems approach ..... 31
3.2 Simulation ..... 31
3.3 Groundwater basin modelling ..... 33
3.4 Mathematicai background ..... 34
3.5 Developing an asymmetric grid ..... 36
3.6 Digital computer solution ..... 37
3.7 Varamin groundwater simulation model ..... 44
3.7.1 Construction of nodal network ..... 44
3.7.2 Preparation of data ..... 48
Transmissivity ..... 48
Storage coefficient ..... 50
water table data ..... 52
Overall groundwater balance ..... 53
Polygonal net deep percolation ..... $6!$
Ground surface elevation ..... 61
Elevation of the impervious base ..... 62
Elevation of the impervious base at the mid-point of the flow path ..... 62
Elevation of the drainage base ..... 62
3.7.3 Calibration of the model ..... 63
4. Agricultural planning ..... 65
4.1 The model ..... 66
4.2 Results ..... 72
4.3 Conclusions ..... 79
4.4 Subregional agricultural production patterns ..... 82
5. Developing a linear programming test model ..... 87
5.1 The test model ..... 87
5.2 Results obtained from the test model ..... 91
6. Developing the comprehensive linear progranming model ..... 97
6.1 Objective ..... 98
6.2 Activities ..... 98
6.3 Constraints ..... 99
7. Calculating costs of activities ..... 106
7.1 Polygonal costs of agricultural production ..... 106
7.2 Polygon cost of surface water ..... 107
7.3 Polygonal cost of well water ..... 118
8. Calculating resource constraints and coefficients ..... 125
8.1 Surface water constraints ..... 125
8.2 Groundwater constraints ..... 125
8.3 Maximum water demand of the polygons ..... 125
8.4 Conveyance and field irrigation losses ..... 127
8.4.1 Conveyance losses in main canals and laterals ..... 127
8.4.2 Percolation losses downstream of farm group inlet ..... 131
8.5 Linear programing matrix ..... 136
9. Procedure in using the models ..... 138
9.1 Determining the maximum river discharge ..... 139
9.2 Determining the maximum groundwater abstraction ..... 141
9.3 Selecting a land-use policy ..... 142
9.4 Schematic of the computer studies ..... 144
10. Resuits obtained from the modelling studies ..... 145
10.1 Water supply scheme No.l ..... 145
10.1.1 Optimal solution ..... 145
10.1.2 Testing the optimal solution for its technical feasibility ..... 148
10.1.3 Cost of water supply ..... 152
10.1.4 Shadow prices of the used constraints ..... 158
10.1.5 The water supply solution and land allocation policies ..... 160
10.1.6 Economic consequences of the hydrological adjustments ..... 163
10.1.7 Sumbary ..... 163
10.2 Water supply scheme No. 2 ..... 164
10.2.1 Solution ..... 164
10.2.2 Testing the solution for its technical feasibility ..... 167
10.2.3 Cost of water supply ..... 167
10.3 Water supply scheme No. 3 ..... 171
10.3.1 Simulating river flow cycles ..... 171
10.4 Water supply scheme No. 4 ..... 174
10.4.1 Simulating artificial recharge ..... 174
10.5 Water supply scheme No. 5 ..... 176
10.5.1 Solution ..... 176
10.6 Water supply scheme No. 6 ..... 180
10.6.1 Solution ..... 180
10.7 Water supply scheme No. 7 ..... 183
10.7.1 Solution ..... 183
10.8 Water supply scheme No. 8 ..... 186
10.8.1 Simulating river flow cycles ..... 186
10.9 Water supply scheme No. 9 ..... 189
10.9.1 Optimal solution ..... 189
10.9.2 Testing the optimal solution for its technical feasibility ..... 192
10.9.3 Adjusted solution ..... 200
10.9.4 Cost of water supply ..... 200
10.9.5 The water supply solution and the irrigated area ..... 204
10.9.6 Shadow prices of farmers in the different polygons ..... 206
10.9.7 Economic consequences of the hydrological adjuscments ..... 208
10.9.8 Summary ..... 208
10.10 Water supply scheme No. 10 ..... 209
10.10.1 Solution ..... 209
10.10.2 Testing the solution for its technical feasibility ..... 212
10.11 Water supply scheme No. 11 ..... 215
10.11.| Simulating river flow cycles. ..... 215
10.12 Water supply scheme No. 12 ..... 219
10.12.1 Simulating artificial recharge ..... 219
10.13 Parametric programing ..... 222
10.13.1 Schemes Nos. 13 to 17 ..... 222
11. Adjusting the water supply solutions to overcome monthly river discharge deficiencies ..... 228
12. Discussion ..... 233
12.1 Strong and weak points of the applied techniques ..... 233
12.2 Comparison of the feasible solutions obtained ..... 235
12.3 The models as tools for further planning ..... 238
Symbols ..... 241
Authors index ..... 243
Subject index ..... 244
References ..... 247

## 1. Introduction

In a semi-arid region, where water resources are not only limited, but where quantities of surface water are highly variable; planning an efficient irrigation system is a complex undertaking, involving many scientific, engineering, socioeconomic, and management aspects. Basic to this problem is making an optimal joint use of the region's surface water and groundwater resources.

After a region's water resources have been assessed, the problem is not merely how to distribute the water, but how the resources can best be managed. If groundwater basins are overpumped, water tables will drop and may eventually reach a depth where the cost of groundwater recovery becomes prohibitive. Or, if salty groundwater occurs in adjacent areas or in overlying or underlying aquifers, it may intrude into the pumped aquifer and eventually reach the wells.

In areas irrigated with surface water, the inevitable water losses may cause the water table to rise and the land may become waterlogged. If this happens in arid areas, the soil may become salinized; the land may then eventually have to be abandoned, or costly measures must be taken to reclain it.

Problems of this kind are numerous in semi-arid and arid regions: the quality of well water in parts of Pakistan is rapidly deteriorating; water tables in parts of India are rising due to inefficient use of surface water for irrigation, and are falling in other parts due to excessive pumping from wells; the water table in recently reclaimed desert soils along the Nile is rising at ate of 4 m a year, and has reached the land surface in some places where not long before it had been 20 m deep (SCHOLZE and DE RIDDER, 1974). If these all too familiar problems could be quantitatively appreciated and minimised in the planning stage of an irrigation project, much would be gained.

With both surface water and groundwater available for irrigation, the two sources can be used in an almost infinite number of combinations. But irrigation water must be supplied economicaliy, and here we face an economic optimization problem. Basic to this problem is to find, under certain given activities and constraints, the most economic solution, making an optimal use of the two water sources.

In this publication we present a methodology that we developed to enable us to study these problems more objectively. It was worked out some ten years, ago, when we were members of an FAO-UNDP team in Iran, working on the project named "Integrated Planning of Irrigated Agriculture in the Varamin Plain".

At that time, the technique of modelling was not yet as advanced as it is today and among the optimization techniques then available, linear progranming was the one most commonly applied. It was for this reason, and because a standard computer progranme of the Simplex method was available, that we applied linear programming to the problem of the optimal joint development and supply of surface water and groundwater.

We also constructed a mathematical groundwater basin model for the region. This model was essentially the same as that developed a few years earlier in California (CHUN, WEBER, and MIDO, 1963; WEBER, PETERS, and FRANKEL, 1968). We then linked the linear programing model to the groundwater model, grafting one upon the other. In this way we could immediately test each linear programming solution of irrigation water supply on the groundwater model to find out the effect it would have on the water table. If unacceptable changes in the water table were produced, the solution was adjusted as many times as was necessary to bring the water table changes within acceptable limits. Several technically feasible water supply solutions were thus obtained, giving decision-makers a firmer basis on which to choose the direction in which the ultimate solution to the problem must be sought.

When considering our approach, the reader may come across some weak links in the chain. We did not, for example, use linear programning for planning agriculturai production in the various sub-areas into which we had divided the project area in order to develop the groundwater model; we used the cropping patterns produced by the project. Likewise, we did not apply this technique to assess the impact that water-deficient years would have on the agricultural production plan. Further, the study is based on obsolete cost values, i.e. values that were available to us in 1966 when we were working on the project; with updated cost values the computer calculations can easily be repeated. Then, although the results of the study clearly revealed that for each water supply solution a number of farmers would have to be resettled, we did not further specify the social and economic implications of such resettlement problems. Neither did we have sufficient time to make a cost analysis of the artificial recharge measures necessary for a proper management of the groundwater resources. Finally,
the water availability was calculated on an annual basis instead of a fortnightly one as is required in irrigation. The reason for this was that at the time we were working on the models many data were net yet available, while the historic daca on irrigation water distribution and supply were too scanty to work with such a small time step.

There is no doubt that these shortcomings reduce the practical value of this study. In spite of its imperfections, most of which can largely be overcome by further investigations, we hope that our approach may be of some use to planning teams facing problems of the optimal joint development and supply of surface water and groundwater for irrigation.

## 2. Description of the Varamin Plain

### 2.1 Location and extent

The Varamin Plain is situated on the southern slopes of the Elburz Mountain Range, some 40 km southeast of Teheran. Its area is roughly $1,200 \mathrm{~km}^{2}$ and its altitude is between 800 and $1,100 \mathrm{~m}$ above mean sea level (Fig. 1 ).

### 2.2 Geomorphology

The Varamin Plain is the alluvial fan of the Jaj Rud, a perennial river which rises in the Elburz Mountains northwest of Teheran. The average slope of the land is 6 m per km , but at the apex of the fan it is approximately 13 m per km and at the lower end, near the desert, 3 m per kme (Fig. 2).

An important tributary, the Damavand Rud, joins the main river some 15 km upstream from where the Jaj Rud debouches into the Plain. At the apex of the alluvial fan the Jaj Rud diverges into a large number of branches and its sediment-carrying capacity thus diminishes. Coarse and very coarse sediments therefore occur at the apex, while farther downstream the sediments gradually become finer, passing into loam and silt in the lower parts of the fan.

The Pishiva Hill, a prominent complicated tectonic structure, protrudes into the alluvial fan from the east-southeast, dividing it into two parts. This anticlinal ridge, though partly eroded by the Jaj Rud, acted as a barrier to the river and caused the somewhat irregular shape of the alluvial fan.

The Jaj Rud is the Plain's only source of surface water. Some minor creeks descend from the foothills in the north, but they only carry water during heavy rain storms.


Fig.1. Location of the Varumin Plain and Juj Rud watershed.


Fig.2. Iopographical map of the Varamin Plain.


Fig. 3. Geologic map of the Varamin Plain (ANONYMOUS, 1964).

### 2.3 Geology

The Varamin Plain represents an intermontane basin that is bounded on the north by the Elburz Range and on the south by the Siah Kuh Range. The basin consistis of down-warped Palaeozoic and Mesozoic sediments and Eocene volcanics, which are covered with Young Tertiary and Quaternary deposits of the Jaj Rud (Fig. 3). These river deposits, which are mainly Pleistocene in age, are more than 300 m thick in some places and, owing to their sandy nature, represent an important reservoir of good quality groundwater.

At its base this reservoir is underlain by clayey Miocene and Mio-Pliocene sediments, which outcrop in the Elburz foothills in the north, the Siah Kuh Range in the south, the Pishva Hill, and at some places on the Tertiary Plateau in the west. To the east there are no such specific boundaries, but there are outlets to the desert in the southeast (Dasht-e-Kavir) and to the Eyvankey Plain in the northeast.

From a tectonic viewpoint, the Varamin Plain is made up of two synclinal depressions separated by an asymmetrical anticline, the Pishva Hill, which is bounded on the south by a fault. Although this mountain spur seems to terminate at the town of Pishva, it can be traced in the sub-surface in a northwesterly direction (Fig.4) ,

The tectonic movements which gave rise to the folding and faulting of the sediments date mainly from the Pliocene and continued during the Pleistocene. This is confirmed by the dipping and faulting observed in the Hezardareh Formation (Bakhtiary conglomerate), which is Plio-Pleistocene in age. Earthquakes form evidence that the tectonic movements have not yet ceased.

Geoelectrical investigations and exploratory well drilling have shown that the thickness of the Quaternary river deposits above the buried Pishva ridge is only 100 m at some places, whereas it is 200 to 300 m in the synciinal depressions to the north and south of the ridge (Fig.3).



### 2.4 Land resources

A preliminary soil survey had been made by consultants several years prior to the F.A.O. project, but it was felt wise to make a more detailed survey of the soils and land resources of the Plain: This new survey was done by the Soil Institute of Iran, working in collaboration with project team members.

The Varamin Plain comprises mainly alluvial soils derived from the southern slopes of the Elburz Mountains and deposited by the Jaj Rud. There are also small areas of colluvial materials washed off the hills bordering the Plain. The hills to the north of the Plain consist of Miocene, gently folded, interbedded layers of consolidated marls, clays, silt, and gypsum.

The soils of the Varamin Plain do not show great genetic differences in their profile development. Moreover, they have been disturbed for centuries by agricultural activities and irrigation. Consequently a regular pattern of soil characteristics can hardly be found.

The natural landscape is the only reliable basis on which the Varamin soils can be mapped and classified. Hence the Eollowing physiographic units were distin-. guished: piedmont fans, old Jaj Rud fans, recent Jaj Rud fans, upper Plain, middle Plain, lower Plain, and foothill slopes.

The soils on the different fans are shallow to very shallow and they consist of gravelly clay loam, loam over gravels, gravelly sand over cobbles and gravels. In the upper Plain deep grey soils occur, consisting of sandy clay loam, to clay loams with little lime. Since the water table is very deep ( 60 m ) there is no capillary salinization.

In the middle Plain deep, grey-brownish soils occur, made up of clay loam to silty clay loam with lime accumulation and at some places gypsum at 70 cm .

The lower Plain has deep solonchak soils, consisting of fine sandy clay loam to silty clay loam with salt crusts. Where the subsoil is clayey, the soils are poorly drained. A major part of the lower Plain has saline soils, made up of deep, brown, moderately saline clay loam to silty clay loam with intercalced gypsum. The foothill slopes have deep gypsiferous soils of brown clay and clay loam that is strongly saline. At some places these soils are eroded or very shallow: gravelly soils over pure gravel are comon on these slopes.

The different physiographic units with their corresponding soil types, hectarages,
and area percentages are given in Table 1 . It can be seen from this table that the deep, grey-bromish soils in the middle Plain cover an area of roughly 40,000 ha, or 34 per cent of the surveyed area. Another 20,000 ha of good, slightly lighter soils occur in the upper Plain. All irrigated soils have a higher compaction and lower porosity than the non-irtigated soils, but the porosity and aeration of the irrigated, grey-brownish soils is still fairly good.

TABLE 1. The soils of the Varamin Plain

| Physiographic unic | Phase | Area <br> (ha) | Area (Z) | Description |
| :---: | :---: | :---: | :---: | :---: |
| Piedmont fans | shallow | 1,200 | 1.0 | Pale, gravelly clay loam aver gravels (colluvial soil) |
| Old Jaj Rud fans | shallow | 2,200 | 1.9 | Brownish grey, gravelly clay <br> loams to loams over gravels |
| Recent Jaj Rud fans. | very <br> shallow | 15,000 | 12.8 | Grey, gravelly sand over cobbles and gravels |
| Upper Piain | deep grey <br> shallow | 19,950 | 17.1 | Sandy clay loam to clay loams with little lime Gravelly, eroded salide phase |
| Middle Plain | deep grey brownish <br> gypsiferous subsoil | (40,450 | 34.6 | Clay loam to silty ciay <br> loam with lime accumulation <br> Gypsum at 70 cm , saline phase, slightly saline |
| Lower Plain | deep solonchak <br> poorly drained | 27,950 | 23.9 | Fine sandy clay loam to silty clay loam, salt crust <br> Clayey subsoil, lot permeability |
|  | deep saline | 6,230 | 5.3 | ```Deep brown, moderately saline clay loam to silty clay loam, gypsiferous``` |
| Foochill slopes | deep gypsiferous | 3,220 | 2.8 | Brown clay to clay loam, strongly saline (colluvial soils) |
|  | eroded, shallow |  |  | Gravelly soils over gravel |
| Miscellaneous | various | 700 | 0.6 | River beds, rocks, mounds |
| TOTAL |  | 116,900 | 100.0 |  |

The land of the Varamin Plain was classified. Six land classes were distinguished, each class divided into a number of sub-classes. The six main classes are presented in Fig. 5 and Table 2.

TABLE 2. Land classification of the Varamin Plain

| Land Class | Total area (ha) | Percentage | Description |
| :---: | :---: | :---: | :---: |
| 1 | 32.750 | 28.0 | Arable, good irrigable lands |
| II | 22,650 | 19.4 | Arable, moderately good irrigable lands |
| III | '12,600 | 10.8 | Arable, marginal irrigable lands |
| IV | 15,900 | 13.6 | Non-arable, irrigable under special conditions and crops |
| $V$ | 32,400 | 27.7 | Non-arable, non-irrigable under present conditions |
| vi | 700 | 0.6 | Non-arable, non-irrigable |
| TOTAL | 117,000 | 100.0 |  |

The areas suitable for agricultural development are those of Class I and Class II and small areas of Class III. As can be seen from Fig. 5 , these lands are located in the central part of the Plain. Of the cotal irrigable land, roughly 60,000 ha, or 50 per cent of the total investigated area, are regarded as suitable for developing irrigated agriculture.

The unsuitable land of Class LV, Class $V$, and Class VI occurs in the fringe areas, particularly in the south. These lands have severe topographical or salinity problems or its soil is very shallow and gravelly.

Table 3 shows the cotal area under cultivation in two different years and the main crops that were grown during those years.

This table clearly reflects the water deficiency in the Plain. In a normal year, when the river flow is 342 million $\mathrm{m}^{3}$, only 62 per cent of the land suitable for irrigation is cultivated. The year $1966 / 67$ was a dry one and less than half of the irrigable land could be used. It should be noted, however, that this low valoe was also partly due to che filling of the Farahnaz Pahlavi Dam at Latiyan.

TABLE 3. Total area under cultivation and the main crops grown in 1957 and 1966

| Year | Total cultivated area (ha) | Area of crops (in ha) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Wheat/ barley | cotton | Melon | Fruit/ vegetables |
| 1957 | 37,200 | 25,200 | 7,800 | 2,400 | 2,600 |
| 1966 | 29,226 | 18,857 | 4,706 | 2,365 | 4,300 |

### 2.5 Climate

The climate of the Varamin Plain corresponds, in general, with that of the Central Iranian Plateau. It may be defined as continental, semi-arid to arid.

The summers are dry and hot. The months of July, August, and September are the driest and the temperature may rise to more than $40^{\circ} \mathrm{C}$, with $44^{\circ} \mathrm{C}$ as the absolute maximum.

Most of the rain falls in winter and early spring, but it is highly variable, not only from month to month, but also from year to year. The winters are cold and the temperature drops to below zero, with $-21^{\circ} \mathrm{C}$ as the absolute minimum.

In the summer the relative air humidity may be as low as 20 per cent at mid-day, rising to 63 per cent by sunrise, with an average of 44 per cent.

The total evaporation, based on measurements in a Class A pan, averaged $1,979 \mathrm{~mm}$ a year over the period 1960 through 1968 , its lowest value was $1,628 \mathrm{~mm}$ in $\{968$ and its highest $2,227 \mathrm{~mm}$ in 1961. During the summer months of June and July, evaporation may reach 425 mm a month.

Some characteristic climatic data obtained from the Varamin meteorological station and the newly established station on the experimental field at Dehvin just north of Varamin are given in Tables 4 and 5.

From Table 5 it can be seen that there is a considerable water deficit in spring, autumn, and especially in summer, due to the high evaporation rates.

It is obvious that irrigation is vital for crop growing in the Varamin Plain.

### 2.6 Surface water resources

Irrigation, using water from the Jaj Rud, has been practised in the Varamin Plain for centuries. This river, which is the only major surface water resource available, rises in the Elburz Range, north of Teheran. Its watershed is $1,892 \mathrm{~km}^{2}$, of which at present only $692 \mathrm{~km}^{2}$ are controlled by the Farahnaz Pahlavi Dam. This dam is located near the village of Latiyan, some 38 km upstream

TABLE 4. Precipitation at varamin during the period 1957-1969 (in mm)

| Year | J | $F$ | M | - A | N | J | J | A | S | 0 | N | D | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1957 | - | - | - | 31 | 10 | 14 | 0 | 0 | 0 | 29 | 37 | 16 | - |
| 1958 | 20 ; | - | 33 | 13 | 6 | 2 | 0 | 0 | 1 | 1 | 6 | 23 | $\cdots$ |
| 1959 | 56 | - | - | 11 | 19 | - | 0 | 0 | 0 | 2 | 34 | 2 | - |
| 1960 | 7 | 3 | 27 | 44 | 0 | 15 | 0 | 0 | 0 | 0 | 26 | 16 | 138 |
| 1961 | 42 | 21 | 19 | 17 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 6 | 111. |
| 1962 | 9 | 50 | $\because-$ | 14 | 5 | 0 | 0 | 0 | 0 | 4 | 4 | 1) | - |
| 1963 | 6 | 7 | 4 | 24 | 36 | 0 | 0 | 5 | 0 | 1 | 5 | 44 | 132 |
| 1964 | 0 | 16 | 6 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 24 | 59 |
| 1965 | 77 | 9 | 20 | 5 | 0 | 3 | 0 | 1 | 0 | 33 | 2 | 3 | 153 |
| 1966 | 3 | 7 | 41 | 19 | 5 | 0 | 0 | 0 | 0 | 28 | - | 6. | - |
| 1967 | 3 | 15 | 9 | 13 | 9 | 0 | 0 | 0 | 0 | 0 | 11 | 8 | 68 |
| 1968 | 9 | 36 | 24 | 37 | 31 | 10 | 0 | 0 | 0 | 6 | 33 | 13 | - |
| 1969 | 106 | 8 | 32 | 25 | 7 | 0 | 0 | 0 | 9 | 19 | 6 | 10 | 233 |
| Mean | 28.2 | 17.2 | 22.6 | 20.2 | 9.8 | 3.7 | 0.5 | 0.5 | 0.8 | 9.5 | 14.0 | 14.0 | 141.0 |

TABLE 5. Some meteorological data from the Dehvin station (1967-1969)

| Month | $\text { Temp }_{\mathrm{C}}$ | Precipitation (mm) | Rel, hum. (3) | Open pan evap. (mm) | $\begin{aligned} & \text { ETP (Penman) } \\ & \text { (mm) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Jan. | 5.3 | 45.0 | 67 | 22 | 13 |
| Febr. | 5.5 | 15.0 . | 71 | 33 | 20 |
| March | 11.8 | 41.6 | 60 | 91 | 64 |
| April | 14.8 | 35.2 | 50 | 150 | 105 |
| May | 21.5 | 10.0 | 46 | 245 | 196 |
| June | 25.1 | 0.7 | 36 | 277 | 222 |
| July | 28.0 | 0 | 40 | 287 | 230 |
| Aug. | 27.5 | 0 | 36 | 289 | 231 |
| Sept. | 23.4 | 3.7 | 36 | 216 | 151 |
| Oct. | 17.6 | 12.0 | 50 | 146 | 102 |
| Nov. | 12.2 | 19.0 | 63 | 69 | 41 |
| Dec. | 7.2 | 6.7 | 60 | 35 | 21 |
| Meap/ Total | 16.7 | 188.8 | 51 | 1,860 | 1,396 |



Fig. 5. Classification of the land (after Soil Institute of Iran, June 1968) superimposed on the polygon network (see Fig. 19).
$\square$
from where the river enters the Plain (see Fig.l) . Approximately 22 km downstream of the dam an important tributary, the Damavand Rud, joins the main river.

At the time of our study; river flow records at latiyan were available for a period of 22 years ( 1946 to 1968 ), but no information existed on the discharge at the site where the Jaj Rud enters the Varamin Plain. Such information was difficult to collect because the river bed near the entrance to the Plain is very wide and instead of a single stream channel it consists of numerous braiding channels.

The discharge of the Damavand Rud just upstream of the confluence was not known either, but during our study a gauging station was erected at this site. Unfortunately, only a limited number of flow measurements (covering less than one year) could be taken here, because during a heavy flood in spring the station was completely destroyed. Downstream of the confluence, at Darvazeh, about 10 km upstream from the river mouth, another gauging station was erected, and a series of flow measurements were taken there (Fig. 1).

A rainfall-runoff relationship was established for the Damavand Rud and with the help of that relation a series of 22 years of. discharges.could be created.

Cor'relation studies of the discharges of the Jaj Rud at Darvazeh and Latiyan were made and it was thus possible to create a series of 22 years of historical river flows at Darvazeh (see Table 6). Note that Mehr 1 st equals September 21 St. The year 1325 corresponds with the Christian year 1946. The Iranian year commences on the Ist Farvardin, which corresponds with the 21 st March.

### 2.6.1 Selecting a discharge distribution

The records of the Jaj Rud flow at Darvazeh cover only 22 years. Consequently the rigorous statistical techniques for testing the goodness of fit of theoretical distributions to large quantities of empirical data could not help in choosing a distribution. The choice therefore had to be made by intuition and conmon sense.

TABLE. 6. Total monthiy and annual discharges of the Jaj Rud at the Darvazeh gauging station
for 22 hydrological years (miliion mif after. VUKCEvIC (l970)

| $\begin{aligned} & \text { MONTH } \\ & \text { YEAR } \end{aligned}$ | MEHR | ABAN | AZAR | DAY | BAH | ESF | FAR | ORD | KHOR | TIR | MOR | SHAH | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1325-26 | 13.45 | 15.73. | 13.45 | 14.41 | 17.65 | 33.30 | 47.00 | 52.71 | 34.60 | 16.20 | 8.86 | 6.53 | 273.89 |
| 1326-27 | 6.66 | 10.21 | 8.27 | 8.27 | 8.58 | 12.75 | 43.98 | 82.82 | 69.42 | 32.60 | 17.86 | 13.18 | 314.60 |
| 1327-28 | 11.20 | 14.10 | 16.41 | 14.47 | 20.58 | 35.58 | 146.37 | 196.59 | 139.68 | 46.34 | 23.22 | 17.57 | 680.11 |
| 1328-29 | 17.68 | 16.72 | 16.04 | 13.11 | 19.59 | 51.74 | 82.82 | 112.92 | 86.16 | 32.27 | 16.90 | 14.22 | 480.17 |
| 1329-30 | 9.25 | 10.55 | 10.55 | 9.23 | 9.23 | 21.20 | 38.62 | 43.66 | 28.18 | 7.53 | 5.19 | 4.18 | 197.37 |
| 1330-31 | 8.61 | 18.32 | 14.10 | 13.76 | 19.28 | 27.97 | 112.95 | 183.20. | 79.12 | 25.23 | 16.23 | 13.90 | 532.67 |
| 1331-32 | - 12.49 | 12.80 | 13.14 | 13.11 | 15.06 | 29.54 | 75.13 | 125.32 | 80.14 | 41.30 | 21.27 | 13.55 | 452.85 |
| 1332-33 | 11.12 | 26.51 | 22.56 | 21.49 | 25.04 | 49.46 | 128.70 | 186.47 | 78.18 | 62.03 | 45.37 | 19.95 | 676.88 |
| 1333-34 | 18.74 | 18.97 | 19.34 | 16.72 | 16.77 | 25.91 | 72.98 | 74.59 | 49.87 | 21.80 | 9.53 | 5.92 | 351.14 |
| 1334-35 | 8.37 | 9.69 | 12.03 | 11.53 | 10.88 | 33.98 | 102.15 | 150.04 | 71.03 | 32,99 | 18.40 | 13.36. | 474.43 |
| 1335-36 | 9.67 | 11.12 | 10.01 | 8.84 | 10.24 | 22.98 | 60.05 | 104.72 | 78.26 | 37.79 | 18.77 | 12.69 | 385.14 |
| 1336-37 | 17.81 | 18.07 | 19.21 | 17.99 | 19.03 | 39.14 | 78.13 | 64.71 | 34.66 | 16.50 | 9.94 | 7.02 | 342.21 |
| 1337-38 | 9.20 | 9.93 | 13.30 | 10.16 | 12.60 | 24.86 | 109.12 | 90.64 | 53.59 | 26.25 | 18.00 | 15.56 | 393.21 |
| 1338-39 | 8.14 | 9.54 | 8.92 | 10.00 | 9.23. | 10.86 | 21.86 | 38.14 | 16.90 | 7.50 | 4.58 | 3.67 | 149.34 |
| 1339-40 | 3.99 | 7.13 | 8.19 | 6.45 | 10.29 | 12.83 | 36.21 | 64.39 | 28.66 | 11.76 | 6.96 | 4.87 | 201.73 |
| 1340-41 | 4.98 | 6.17 | 6.77 | 7.23 | 11.07 | 21.20 | 27.56 | 97.09 | 44.89 | 18.21 | 9.40 | 6.67 | 261.24 |
| 1341-42 | 7.05 | 8.99 | 8.27 | 8.45 | 14.28 | 18.17 | 43.64 | 97.36 | 55.36 | 23.73 | 15.59 | 9.62 | 312.51 |
| 1342-43 | 8.66 | 13.71 | 13.35 | 11.90 | 11.95 | 31.10 | 62.25 | 62.51 | 24.93 | 10.09 | 6.40 | 5.19 | 262.04 |
| 1343-44 | 5. 62 | 6.97 | 7.91 | 7.15 | 13.63 | 44.75 | 59.62 | 92.38 | 48.32 | 20.11 | 8.41 | 7.02 | 321.89 |
| 1344-45 | 8.29 | 13.79 | 9.87 | 8.29 | 16.38 | 25.93 | 40.15 | 63.18 | 36.75 | 16.44 | 8.73 | 5.81 | 253.61 |
| 1345-46 | 12.13 | 14.91. | 10.35 | 11.46 | 10.66 | 17.26 | 28.11 | 59.93 | 30.24 | 9.87 | 6.09 | 4.47 | 215.48 |
| 1346-47 | 8.66 | 9.12 | 9.64 | 7.59 | 9.98 | 32.35 | 81.45 | 92.73 | 153.53 | 41.19 | 14.25 | 7.20 | 467.69 |
| SUT | 221.77 | 283.05 | 271.68 | 249.61 | 312.00 | 622.84 | 1500.85 | 2136.10 | 1322.47 | 557.73 | 309.95 | 212.15 | 8000.20 |
| AVERAGE | 10.08 | 12.87 | 12.34 | 11.35 | 14.18 | 28.31 | 68.22 | 97.10 | 60.11 | 23.35 | 14.09 | 9.64 | 363.64 |
| $1325+621=1946$ |  | $7 s$ | hr | t S | er |  |  |  |  |  |  |  |  |

The log-normal distribution function is widely used in statistical work of this type. The basic justification for this distribution function is the central limit theorem, which states that the logarithm of a variable, which is the sum of identically distributed random variables derived from any distribution with a finite mean and variance, is distributed normally. In this study we made use of the logarithm of the cotal sum of river flows in a hydrological year, starting on the 21st September of one year and continuing until the 20th September of the next. Figure 6 shows a plot of these river flows. It also shows the line of the theoretical log-normal distribution, calculated with the mean and variance. This line matches the points reasonably well.

For the sample coefficient of skemess, which must be equal to zero for a lognormal distribution, a value of 0.08 was found. We could therefore conclude that the assumption of a log-nomal distribution for the river flow at Darvazeh was justified, i.e. it was a fair assumption that such a distribution existed.


Fig.6. Frequency distribution of annual river flows at Darvazeh (log-normal).

In the first instance we were interested in the total annual river flows that have a return period of 20,5 , and 1.67 years, corresponding with an exceedance frequency of respectively 95,80 , and 40 per cent. It can be seen from Fig. 6 that these frequencies refer to river flows of 173,240 , and 370 miliion $\mathfrak{m}^{3}$ a year.

Gumbel distribution

For reasons of comparison we also assumed that the synthesized flows at Darvazeh have a Gumbel distribution, which is a special case of the log-nomal distribution, .

Figure 7 shows a plot of the synthesized data for the gauging station at Darvazeh on what is known as Gumbel paper. The straight line was calculated, using the method of maximum probability: From this diagram the following river flows could be derived:

$$
\begin{array}{ll}
175 \times 10^{6} \mathrm{~m}^{3} \text { a year; return period of } 20 \text { years } \\
245 \times 10^{6} \mathrm{~m}^{3} \text { a year; return period of } 5 \text { years } \\
365 \times 10^{6} \mathrm{~m}^{3} \text { a year; return period of } 1.67 \text { year }
\end{array}
$$

These discharge valties differ only slightly from those obtained from Fig. 6.


Fig.7.' Frequency distribution of annual wiver flows at Dasvazeh (Gumbel).

In considering the results of the above analyses, the following points must be borne in mind:

Firstly, from a statistical viewpoint, the 22-year flow record is very short, so that we do not know whether the river discharges fit any distribution at all. It may well be that over a period of say 50 years, an entirely different distribution would be found.
Secondly, the synthesized discharges of the Damavand Rud may be subject to error, because substantial quantities of river water are used to irrigate crops grown in the valley of the river.
Thirdly, in the 10 km -tract between the gauging station at Darvazeh and the place where the Jaj Rud enters the Plain, substantial quantities of river water percolate into the coarse alluvial materials on the valley bottom. The thickness of this material is about 10 m near Darvazeh, but increases in downstream direction and may attain several tens of metres near the river mouth. These percolation losses are not known because flow measurements cannot be taken in the numerous braiding stream channels that occur in this tract of the river.

Between Darvazeh and Latiyan, where the river flows in a single stream bed, flow measurements have shown that the percolation losses may be of the order of $40 \times$ $10^{6} \mathrm{~m}^{3}$ a year (VUKCEVIC, 1969). It is quite possible that in the wide valley mouth downstream of Darvazeh another $25 \times 10^{6} \mathrm{~m}^{3}$ water is lost by percolation. Hence the total percolation loss between Latiyan and the Varamin Plain may be of the order of $65 \times 10^{6} \mathrm{~m}^{3}$ a year. This quantity of water enters the plain as a subsurface inflow.

Finally it should be noted that the Damavand Rud, which has a watershed of $776 \mathrm{~km}^{2}$, or slightly more than that of the Jaj Rud upstream of Latiyan, is uncontrolled. The dam in the Jaj Rud at Latiyan controls only a part of that river's catchment area, its main purpose being to safeguard Teheran's drinking water supply. The dam can store approximately $100 \times 10^{6} \mathrm{~m}^{3}$, of which $80 \times 10^{6} \mathrm{~m}^{3}$ a year is diverted to Teheran.

In spring when the snow on the Elburz mountains melts and heavy rain sometimes occurs, the largely uncontrolled Jaj Rud may have a very high discharge for several hours. Even though all channels and irrigation canals are then running full, the Plain cannot absorb all this water and part of it is spilled to the desert in the south through the main river branch on the west. Since no other dam in the Jaj Rud is envisaged and the diversion dam in the mouth of the river will only be a low one, a.certain spillage of river water during high discharges is something which will have to be reckoned with in the future as well. How much
river water will have to be discharged to the desert unused is not yet known, but it may vary from approximately $10 \times 10^{6} \mathrm{~m}^{3}$ to $100 \times 10^{6} \mathrm{~m}^{3}$ anmally depending on climatological conditions.

Since the dam at Latiyan is not capable of meeting the ever increasing. water gemands of thè capital, a new dam in the Lar Rud is under construction. The Lar Rud drains a catchment area on the other side of the Elburz divide and runs to the Caspian Sea (Fig. I). Its annual discharge is of the order of $430 \times 10^{6} \mathrm{~m}^{3}$. The Lar water will be transported to the dam reservoir at Latiyan through a tunnel so that, theoretically speaking, a portion of the $80 \times 10^{6} \mathrm{~m}^{3}$ of Jaj Rud water now being taken away from the Varamin Plain might be replaced by Lar water. Yet it is doubtful whether this will ever happen, as Teheran's water supply has first priority and Mazandaran's agriculture (rice) along the coast of the Caspian Sea, which is mainly based on chis water, cannot be jeopardized by diverting substantial quantities of Lar water to other areas.

With more and more surface water entering Teheran, the day may come that a solution must be found for the disposal of its increasing quantities of sewage water. One such solution would be to purify this water and divert all or part of it by a pipe line or canal to the Varamin Plain, a distance of 40 km . The topographical conditions are favourable for gravity transport; starting in downtown Teheran this water could reach the upper part of the Plain by gravity. At this moment nothing can be said about whether such a plan will ever be realized.

It is obvious that there is still a great deal of uncertainty as to the quantities of surface water that will be available to the Varamin Plain in the future.

$$
\therefore
$$

Under these circumstances, we eventually decided to use the following quantities of surface water:

$$
\begin{array}{ll}
150 \times 10^{6} \mathrm{~m}^{3} \text { a year; return period of } 20 \text { years } \\
220 \times 10^{6} \mathrm{~m}^{3} \text { a year; return period of } 5 \text { years } \\
340 \times 10^{6} \mathrm{~m}^{3} \text { a year; return period of } 1.67 \text { year. }
\end{array}
$$

These quantities, which had been obtained during earlier preliminary studies, differ only slightly from those derived from Figs. 6 and 7.

### 2.7 Groundwater resources

### 2.7.1 Groundwater in storage and present recovery

Underneath the entire Varamin Plain lies a huge body of groundwater. Its estimated volume is $15 \times 10^{9} \mathrm{~m}^{3}$. Not all of this water, however, is of good quality. Along and beyond the Plain's boundaries it is very salty and quite unsuitable for irrigation.

The volume of good quality groundwater in storage ( $\mathrm{Class} \mathrm{C}_{2} S_{1}$ ) is estimated to be $12 \times 10^{9} \mathrm{~m}^{3}$, which is 34 times the average annual flow of the Jaj Rud at Darvazeh. Although this is a substantial quantity, it does not mean that unlimited use can be made of it. A heavy overdraft could cause the inflow of salty groundwater into the basin, especially along the western limits.

Since ancient times groundwater has been recovered in the Varamin Plain by qanats. A qanat is an underground tunnel, constructed through the alluvial material, which transmits water by gravity from beneath the water table to the ground surface (Fig.8).


Fig.8. Scheme of a qanat.

The tunnel has a gentle slope to prevent orosion and collapse, and is constructed in an upslope direction from the point that has been selected as the qanat outlet. The cross section of the tunnel is usually elliptical with a height of about 1.2 m and a width of about 0.8 m . Wherever possible, the tunnel is unined, but in areas of weakly consolidated sediments baked clay rings are used to avoid roof and wall collapse. Once the direction of the tunnel has been determined, a mother well is dug at the upslope end to determine the depth to the water table and the subsurface sedimentary material. Tunnel construction starts in upslope direction from the selected outlet. To provide ventilation for the workers in the tunnel and to facilitate the removal of spoil, a series of vertical shafts, about 20 m apart, are dug along the line of the tunnel.

The major part of the tunnel is constructed above the water table and when at a cercain moment the water rable is struck, upslope construction continues for some distance below the water table till the mother well is reached. In a qanat we can therefore recognize a relatively short "wet" section, which is in fact an underground drain into which groundwater seeps. This is the water-producing section of the qanat. The downslope section is the "dry" section, which merely acts as the transportation section of the qanat. Whereas the "wet" section is only a few tens of metres long, the "dry" section may extend over several or even many kilometres. Qanats as long as 30 to 50 km and even 70 km have been reported in Iran (BEAUMONT, 1968, 1971, 1973).

During a hydrogeological study of the Varamin Plain, EMADI (1966) found a total of 266 qanats in the region. The depth of their mother wells varies from less than 10 m to more than 100 m , the majority having depths between 10 and 20 m ( 161 qanats) and 20 to 30 m ( 64 qanats).

During Emadi's study the discharges of these qanats were also measured. The measurements were taken seasonally, but in 30 qanats the discharge was measured monthly. Figure 9 shows the frequency distribution of the qanat discharges for the year 1964. As can be seen from this figure there were 96 qanats which did. not yield any water in that year. The majority of the productive ganats (134) had yields that ranged from less than $101 / \mathrm{sec}$ up to $301 / \mathrm{sec}$. A small number of ganats had higher yields; three qanats yielded more than $1001 / \mathrm{sec}$.

The discharge of a qanat is a function of the length of its water-bearing section and the transmissivity of the aquifer in that section. It is obvious that fluctuations in the height of the water table due to recharge or discharge of the


Fig.9. Frequency distribution of qanat discharges in 1964.
Total number of qunats: 266 .


Fig.10. Monthly discharges of eight qanats and monthly rainfall in 1964/1965.
groundwater basin will cause variations in the discharge of a qanat. Such variations are obvious from Fig. 10 , which shows the monthiy discharges of 8 qanats for the year 1964/1965.

One would be inclined to think that there is a marked correlation between qanat discharge and rainfall. It is, however, doubtful whether rainfall contributes much to the recharge of the basin because the quantities of rain are so small. Of greater importance for the recharge of the groundwater is the river water, which becomes available during the wet season and is used to (pre-)irrigate the lands. The portion of water that percolates causes the water table to rise and consequently the qanat discharge increases. Several qanats therefore show an anomalous discharge pattern with peaks in the summer months and low yields in spring and autumn. Heavy pumping from deep wells located in the vicinity of qanats may cause the water table to drop; if so, the discharge of the qanat will decline and may eventually cease.

Since more deep wells are sunk in the Plain every year, it is not surprising that this is having a disastrous effect on the qanat yields (see Table 7).

TABLE 7. Well and qanat discharges in the period 1963/64 to 1969/70 (million $\mathrm{m}^{3}$ )

| Year | Qanats |  | Shallow wells |  |  | Deep wells |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | number | yield |  | number | yield |  | number |
| $1963 / 64$ | 185 | 150 | 22 | 4 | 133 | 109 |  |
| $1964 / 65$ | 165 | 105 | 25 | 4 | 195 | 125 |  |
| $1965 / 66$ | 116 | 71 | 25 | 4 | 195 | 143 |  |
| $1966 / 67$ | 115 | 52 | 42 | 8 | 209 | 201 |  |
| $1967 / 68$ | 73 | 22 | 42 | 9 | 210 | 178 |  |
| $1968 / 69$ | - | - | 44 | 11 | 219 | 193 |  |
| $1969 / 70$ | - | - | 48 | 12 | 253 | 226 |  |

The values shown in this table have been plotted in Fig.li together with the annual river flows and the annual rainfall on the Plain. It can be seen from this figure that the total annual qanat discharges do not correlate with the annual river flow or the annual precipitation. The reason for this is that the increasing extraction from deep wells is causing a decline in the water table and in the yield of the qanats, many of which have dried up.


Fig.11. Anmuat discharges of river, qanats, and deep wells, and rainfall.

There is no doubt that the qanat is one of the outstanding engineering works of the past. If we consider the long time it takes to construct a qanat (a matter of many years) and the volumes of material removed solely by manual labour, the human achievement is impressive.

Qanats have certain advantages and disadvantages. An advantage is the important role they play in controling the groundwater table; especially in the middle and downstream parts of the Plain. Here the qanats are approximately 10 m below the surface and in the downstream areas they are even shallower. Obviously the fairly dense network of qanats acts as a subsurface drainage system that prevents the water table from rising to the surface during the wet part of the year. This explains why in these parts of the Plain there is no soil salinization due to capillary rise from a shallow water table. The abandonment of the qanats would mean that, unless the Plain's water resources are properly managed, such problems could occur in the future.

One of the disadvantages of the qanats is their variable, non-regulated, and noncontrolled flow. A qanat flows continuously and especially during the wer season when the discharge is high and the demand for irrigation water is low, most of the water runs to waste. During the growing season, when the demand for water is high, the yield of many qanats gradually declines.After a number of consecutive dry years, the declining water table causes many qanats to dry up completely. Qanats need regular repair when floodings cause then to collapse, and if their flow ceases they must be extended upslope to restore the yield, Compared with a modern tube well, a qanat is an unreliable source of water.

Many qanats are therefore nowadays being abandoned and no new ones are being made; for the same price or slightly more a tube well can be drilled.

In our study we initially retained the qanats, but for the reasons outlined above we later discarded them and now consider tube wells the only device for groundwater recovery and water table control.

### 2.7.2 Groundwater quality

Although the quality of the groundwater in the Plain is good to fair, there are marked differences in both horizontal and vertical directions (see Figs. 12 and 13). As can be expected, the best quality groundwater occurs in the head of the alluvial fan, where the electrical conductivity of the water is as low as 400 to 500 micromhos $/ \mathrm{cm}$. Salty to very salty groundwater with an electrical conductivity of 5,000 to 10,000 micromhos $/ \mathrm{cm}$ occurs near the perimeter of the basin.

The shallow groundwater has a slightly higher salt content than the deep groundwater. Some deep wells in the lower part of the basin have revealed the presence of saline groundwater in the upper 65 to 130 m of the basin sediments, whereas below these levels remarkably fresh groundwater occurs. This water has a chloride content of only 30 to $60 \mathrm{mg} / \mathrm{l}$, or as 10 w as that of the groundwater in the head of the fan, some 45 km farther upstream.

The reason for the poorer quality of the shallow groundwater is that farmers usually over-irrigate their lands, resulting in a downard flow of water to the water table. The salts accumulated in the upper soil layers are (partly) washed down to the underground where they join the shallow groundwater, which thus becomes salinized.


Fig. 12. Electrical conductivity of the shallow groundwater ( $E C$ in micromhos/cm).


Fig. 13. Electrical conductivity of the deep groundwater ( EC in micromhos/cm)

The reason why the deep groundwater is less salinized than the shallow groundwater is because of the annual recharge at the head of the alluvial fan. In spring the high river discharges cause floodings of these upper, gravelly parts of the fan. A portion of this water percolates thtough these pervious materials to the water table. From here the groundwater flows downslope through the deep aquifers and becomes only slightly salinized.

The chloride content of the groundwater varies-from $20 \mathrm{mg} / 1$ at the head of the alluvial fan to $2000 \mathrm{mg} / 1$ along the western limit of the basin and at the foot of the Siah Kuh Range in the south and southeast. However, in the greater part of the basin the chloride content is less than $250 \mathrm{mg} / \mathrm{l}$ but beyond this limit it increases rapidly to the above high values.

The sodium adsorption ratio (SAR) of the groundwater varies from less than 1.5 in the head of the alluvial fan to 20 and 25 along the western limit of the basin and in the southern desert areas. In the, greater part of the basin, however, the SAR values are less than 10 , which means low sodium hazard.

The electrical conductivity and SAR values of the groundwater are commonly used to decide whether the groundwater is suitable for irrigation (RICHARDS., 1954). The classification made according to this system is shown in Fig. 14 . It can be seen that the quality of the groundwater in the greater part of the basin is good to fairly good (Class $C_{2}-S_{1}$ : medium salinity and low sodium). Towards the peripheral areas of the Plain the quality of the groundwater gradually becomes poorer, and zones of groundwater Classes $C_{3}-S_{1}, C_{4}-S_{1}, C_{4}-S_{2}, C_{4}-S_{3}$, and $C_{4}-S_{4}$ are found as one moves towards the boundaries of the Plain. The worst quality $\left(C_{4}-S_{4}\right.$ : very high salinity and very high sodium) is found in the most western part of the Plain beyond the main branch of the Jaj Rud. Where the soil has a good permeability and the water table is deeper than 5 m , the groundwater of Class $C_{2}-S_{1}$ and that of $C l a s s C_{3}-S_{1}$ can be used for irrigation without causing salinity problems. This means that practically all groundwater in the Plain is suitable for irrigation. In the lower part of the Plain the deeper aquifers should preferably be used, as they contain groundwater of good quality $\left(C_{2}-S_{1}\right.$ and at some sites even $\left.C_{1}-S_{1}\right)$.


Fig.14. Classification of the groundwater.

### 2.8 Present system of irrigation water supply

In the absence of a main diversion structure at the apex of the alluvial fan, the river water has, for centuries, been distributed to different parts of the Plain by simple man-made diversions. There are some 65 natural channels in the river where it enters the Plair and to supply certain areas with water, labourers use shovels to make temporary dams in some of the channels and clear previous dams out of other channels through which water is then conveyed. The distribution of river water is organized by water users associations, of which there are four in the Plain: Bahnam Pazouki in the northwestern part of the region, Bahnam Sukhteh in the northeastern part, Bahnam Vasat in the middle part (around Varamin town), and Bahnam Arab in the south-eastern part. Such de facto water users' associations have been in existence for centuries throughout Iran and are governed by Moslem customary water law (ANONYMOUS, 1967). The discharge unit used for the distribution of water is called "sang". A sang is the quantity of water flowing through a vertical cut measured by a special brick unit with a capacity of a certain number of hours per day or week. At the mouth of the river 1 sang corresponds to an average of 12 litres a second. At the Mashad-Teheran road in the northern part of the Plain 1 sang corresponds to 10 litres a second.

River water is distributed over the Plain on the basis of:
the availability of water in the river
the water rights of the villages
the cultivated area and the crops grown.

In spring, when the discharge of the river is high, all channels and man-made canals run full and the whole Plain is supplied with river water. Many villages have their own public reservoir, which is then filled. In periods of drought the water from these reservoirs is used for domestic purposes.

During summer and autumn when the river discharge diminishes, the available water is distributed in accordance with the water rights of the villages. Of the 203 villages in the Plain there are 125 that have water rights. Six times
a year the water rights change depending on the discharge of the river:

21 March - 15 June 848 sang is distributed over all 120 villages
15 June - 21 September
21 September - 21 October

21 October - 5 November
5 November - 21 January
21. January - 20 March

848 sang is distributed over all 120 villages
319 sang is distributed over 54 villages
293 sang is distributed over 54 villages
398 sang is distributed over 61 villages
538 sang is distributed over 90 villages
453 sang is distributed over 60 villages

These quantities refer to a particular year when there were about 160 million $\mathrm{m}^{3}$ river water available for distribution. Some villages, which depend entirely on the river for their water supply, receive the same quantity all the year round. other villages, which possess a qanat or a deep well, receive the full quantity of water in spring, but as the river flow diminishes they receive less, and in late summer and autumn when the river discharge is very low they may receive no river water at all. They must then rely entirely on their qanats or wells or must purchase well or qanat water from neighbouring villages that have sufficient water from such facilities.

The maximum capacity of all 65 channels is about 2500 sang. If the river discharge is more than this capacity, the excess water is spilled to the desert through the main Jaj Rud branch on the west.

This spillage amounts to an average of 30 million $m^{3}$ for a normal river discharge; it may be less than 10 million $\mathrm{m}^{3}$ in a dry year and as much as 100 million $\mathrm{m}^{3}$ in a very wet year.

## 3. Digital groundwater basin model

### 3.1 Systems approach

When the project started some ten years ago, a systems approach and the use of digital computer simulations of groundwater basins were rather new methods of problem-solving. They had been developed in California (USA) only a few years earlier and had been successfully applied to the water resources problems of the California Coastal Plain (TYSON and WEBER, 1963; CHUN, WEBER and MIDO, 1963). In 1966 we applied the same approach to the Varamin Plain.

A systems approach aims at a better understanding of a combination of elements which form a complex that can be designated a system. Essentially, this approach tries to distill the essence of a complex physical entity or system, to describe its structure, and co explain its.internal cause-and-effect relations. A system implies a series of interrelated objects, actions, or procedures.

In regarding the planning of irrigated agriculture in the Varamin Plain as a system, we faced the fact that it consists of many different subsystems (see Fig. 15). Here, we shall only concern ourselves with its groundwater subsystem.

### 3.2 Simulation

A systems approach involves the development of models of some sort. The Varamin groundwater basin model is a simulation model on which new pumping patterns, replenishments from percolating surface water, or any other actions that will affect the groundwater subsystem can be simulated,

The model was formulated with arithmetical and algebraic relations, along with non-mathematical logical processes. The model is not intended to solve problems analytically, but to simulate them on a digital computer. Its usage is basically one of iterative trial and adjustment. The procedure is to execute the simulations for alternative plans of irrigation water distribution and supply, examine che consequences of each plan, make alterations to improve the performance of the groundwater subsystem, and then reexecute the simulations for another check on the performance.


Fig. 15. Scheme of the various subsystems of planning irrigated agriculture.

### 3.3 Groundwater basin modelling

The principle of modelling a groundwater basin system is firstly to decompose the system into its various components. For a groundwater basin we have the following input and output components:

Input components:

1. Subsurface inflow
2. Percolation of precipitation
3. Percolation from streambeds and/or irrigation canals
4. Percolation from field irrigation
5. Percolation from artificial recharge.

Output components:

1. Subsurface outflow
2. Discharge into streams and/or drainage ditches and canals
3. Evaporation
4. Evapotranspiration by crops and vegetation
5. Groundwater pumpage.

The continuity concept requires that this system be in balance, i.e. the inflows minus the outflows equal the change in storage in the basin. Modelling a groundwater system means that all the above components have to be properly analyzed and quantified and their interrelationships determined. It also means that the groundwater has to be related to the lithosphere, i.e. the state of the groundwater must be defined: phreatic, confined, or partly confined groundwater; and that the system's parameters must be determined: transmissivity and storage coefficient.

Groundwater basin modelling also involves simplification, which means that the system should not be broken down into its smallest details, this to avoid mathematical difficulties.

When developing the simulation model of the Varamin Basin, we had to simplify the groundwater system; for instance, the aquifer was assumed to be unconfined (twodimensional flow only). Although this is true for major parts of the basin, in some parts, especially where deeper aquifers occur, the water is under pressure.

This may cause validity problems in such areas. Similar problems arose from the poor quality of some of the basic data; for instance, the historical data of irrigation water percolation. Hence, when studying the predicted water table elevations at the end of this publication, the reader should bear in mind that they merely indicate an order of magnitude and not any true values of water table changes.

### 3.4 Mathematical background

The simulation model developed and used for the Varamin groundwater basin is essentially that developed by TYSON and WEBER (1963) for the California Coastal Plain (see also CHUN, WEBER and MLDO, 1963, and WEBER, PETERS and FRANKEL, 1968). A detailed description of this type of model was recently given by THOMAS (1973) to which the reader is referred for a complete understanding of the procedures, mathematical background, and computer programe.

To find a mathematical expression that governs the flow of water through porous media, one can use the equations of motion, continuity, and the laws of thermodynamics. If the flow is laminar and if the inertia force can be neglected, Darcy's. equation can be written as

$$
\begin{equation*}
v=k(x, y, z) F \tag{1}
\end{equation*}
$$

where

```
V = the velocity of the water
F m the applied force
K(x,y,z)=a proportionality constant, known as hydraulic
    conductivity of the medium
```

$$
\begin{equation*}
K(x, y, z)=\frac{C d^{2} \rho g}{n}=\frac{k(x, y, z) \rho g}{\eta} \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{k}(\mathrm{x}, \mathrm{y}, \mathrm{z})=\mathrm{Cd}^{2}= \text { the specific permeability, which is a property } \\
& \text { of the medium alone } \\
& \mathrm{C}=\text { a constant depending on such factors as porosity, } \\
& \text { packing, size, shape, and distribution of the grains } \\
& \mathrm{d} .=\text { the average grain diameter } \\
& \rho \quad= \text { the density of the water } \\
& \mathrm{n}=\text { the viscosity of the water } \\
& \mathrm{g} \quad
\end{aligned}
$$

The force $F$ is equal to the negative gradient of the hydraulic head,

$$
\begin{equation*}
F=-\operatorname{grad} h \tag{3}
\end{equation*}
$$

Substituting Eq. 3 in Eq. 1 yields

$$
\begin{equation*}
v=-k(x, y, z) \operatorname{grad} h \tag{4}
\end{equation*}
$$

If we assume that the density of the water remains constant, and if $\theta$ is the water content of the soil on a volumetric basis (i.e. the ratio of the volume of soil water to the total volume of soil), we may write the continuity equation as

$$
\begin{equation*}
-\operatorname{Div}(v)=\frac{\partial \theta}{\partial t} \tag{5}
\end{equation*}
$$

Combining Eqs. 4 and 5 gives

$$
\begin{equation*}
\operatorname{Div}[K(x, y, z) \operatorname{grad} h]=\frac{\partial \theta}{\partial t} \tag{6}
\end{equation*}
$$

This is an equation with two dependent variables $\theta$ and $h$. To make the equation consistent with respect to one dependent variable, a storage term $S=S(x, y, z)$ is introduced, representing the volume of water that a unit decline in head releases from storage. In doing so, $\theta$ can be replaced by the product Sh and Eq. 6 then takes the form

$$
\begin{equation*}
\operatorname{Div}[K(x, y, z) \operatorname{grad} h]=S(x, y, z) \frac{\partial h}{\partial t} \tag{7}
\end{equation*}
$$

When surface water and groundwater systems are used for irrigation, a cercain quantity $Q=Q(x, y, t)$ of water is introduced into or withdrawn from the aquifer. Since this affects the elevation of the water table, it is highly desirable to be able to predict these variations in elevation $h=h(x, y, t)$ during operations. If we introduce the quantity $Q(x, y, t)$, which represents rainfall infiltration, percolating irrigation water, pumped extractions, evapotranspiration losses, and leakage, Eq. 7 assumes the form

$$
\begin{equation*}
\nabla[K(x, y, z) \nabla h]=S(x, y, z) \frac{\partial h}{\partial t} \pm Q(x, y, t) \tag{8}
\end{equation*}
$$

where $\nabla=\partial / \partial x+\partial / \partial y$ is the differential operator, and the positive sign on $Q$ corresponds to a net upward flow and the negative sign to a net downard flow.

With the help of the continuity equation ( $\mathrm{Eq}, 5$ ), this linear three-dimensional equation can be converted into the following non-linear equation for two-dimensional flow

$$
\begin{equation*}
\frac{\partial}{\partial x}\left[K(x, y, h) \frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[K(x, y, h) \frac{\partial h}{\partial y}\right]=S(x, y, h) \frac{\partial h}{\partial t} \pm Q(x, y, t) \tag{9}
\end{equation*}
$$

This is the basic partial differential equation for two-dimensional groundwater flow, to be solved with appropriate initial and boundary conditions (HALL and DRACUP, 1970).

### 3.5 Developing an asymmetric grid

An approximate solution to the above groundwater flow problem can be obtained by applying the method of finite differences (RICHARDSON, 1910). The basic idea of this method is to replace derivatives at a point by ratios of the changes in appropriate variables over a small but finite interval. This type of approximation is made at a finite number of points and reduces a continuous boundary value problem to a set of algebraic equations (REMSON et al., 1971).

There are several reasons why an asymmetric grid is preferred to a regular one (MacNEAL, 1953). Firstly, project area boundaries are usually irregular. If a regular grid is used, several of the grid points do not fall on the boundary. If the distance between the boundary and its adjacent grid points is significant, special difference formulas have to be developed for points near the boundary (REMSON et al., 1971). A regular grid either needs an excessive number of nodes or introduces an excessive truncation error in some portion of the basin. Secondly, there is the problem of changing the mesh size where abrupt changes occur in the water table gradient. In such areas the mesh size must be reduced if an accuracy comparable. with that in the rest of the aquifer is to be obtained. Finally, historic data on water tables and recharge-discharge data are seldom available at 'regularly spaced distances.

An asymmetric network of polygons overcomes these difficulties. In accordance with the theory of finite differences, the flow rates in a polygonal aquifer portion may be integrated:

$$
\begin{equation*}
\sum_{i} Y_{i, B}\left(h_{i}-h_{B}\right)=A_{B} S_{B} \frac{d h_{B}}{d t}+A_{B} Q_{B} \tag{10}
\end{equation*}
$$

where

| $\mathrm{Y}_{\mathrm{i}, \mathrm{B}}$ | $=\frac{W_{i, B} T_{i, B}}{L_{i, B}} \quad$ is conductance of the path between $\quad M^{3} L^{-1} T^{-1}$ |
| :---: | :---: |
| $w_{i, B}$ | $=$ length of the perpendicular bisector associated with the nodes $i$ and $B$ |
| Ti, B | = transmissivity at midpoint between <br> the nodes i and B $M^{3} L^{-1} T^{-1}$ |
| $L_{i, B}$ | $=$ distance between the nodes i and B . $\mathbf{L}$ |
| $Q_{B}$ | volumetric flow rate per unit area at node $B M^{3} L^{-2} T$ |
| $S_{B}$ | $=$ storage coefficient of polygonal zone <br> associated with node B dimensionless |
| $A_{B}$ | = area associated with node B $\mathrm{L}^{2}$ |
| $h_{B}$ and $h_{i}$ | = water table elevation at node B and i respectively |
| t | = time ${ }^{\text {a }}$ T |

A typical node point, its neighbours, and the polygonal zone associated with it are shown in Fig. 16.

The left-hand side of Eq. 10 is the sumation of subsurface flows between a given area and its surrounding areas. The first tern on the right-hand side represents the rate of change of water storage within polygon $A_{B}$. The second term represents the surface flow rate from the ground surface into or out of the zone of saturation of polygon $A_{B}$.

### 3.6 Digital computer solution

Discretizing the time derivative in Eq. 10 by backward (implicit) differences, one gets

$$
\begin{equation*}
\underset{i}{E}\left(h_{i}^{j+1}-h_{B}^{j+1}\right) Y_{i, B}=\frac{A_{B} S_{B}}{\Delta t}\left(h_{B}^{j+1}-h_{B}^{j}\right)+A_{B} Q_{B}^{j+1} \tag{11}
\end{equation*}
$$



- $\mathrm{T}_{\mathrm{iB}}=$ characteristic transmissivity between polygion $B$ and polygon $i$
$h=$ representative elevation of groundwater surface in polygon

Fig.16. Scheme of Computer Groundwater Basin Model (after. Fowter and Vatantine, 1963).
where the superscript $j$ denotes the time index and the subscript $i$ denotes the set of nodes adjacent to the node under consideration, i.e. node $B$. This implicit numerical integration has the advantage that the magnitude of the time step $\Delta t$ does not depend on a stability criterion,

The solution of $E q .11$ is straightforward if the values of the parameters $Y_{i, B}$ and $S_{B}$ are known a priori and an initial value of the water table elevation $h_{B}(0)$ is substituted. The values of $h_{B}^{j+1}$ are then implicitly determined at the end of a step in time, $\Delta t$. Once determined, these values become the initial water table elevations for the next succeeding. step in time.

To solve the finite differences equations (Eq.ll) a computer is used. The programme is written in Fortran and the procedure is that all the node-to-node subsurface flows (Q values) are calculated first. Then all the storage flows (S values) are calculated. Next all the flows (subsurface, storage, and external flows) are balanced at each node by setting their sum equal to a residual term (RES). The water table elevation at the node is then adjusted by the magnitude of the residual, attenuated by a relaxation coefficient (RELAX):

$$
\begin{equation*}
H_{\text {new }}=H_{\text {old }}+\text { RELAX } * \text { RES } \tag{12}
\end{equation*}
$$

This relaxation technique for asymmetrical networks was first proposed by MacNeal (1953).

After all the nodal water table elevations have been adjusted in this manner, a sum is formed of all the nodal residuals. This sum is compared with a threshold value (ERROR). The threshold value is a tolerance level, e.g. the maximum acceptable sum of flow residuals at any time step. If the sum of the residual values is less than or equal to the threshold value, the calculation of the water table elevations is complete for that time step. Otherwise the calculation is repeated. as many times as is required to reduce the sum of the residuals to a value less than or equal to the threshold value.

As to the relaxation coefficient (RELAX) it should be noted that the product of the residual term (RES) and the relaxation coefficient results in a change in water table elevation $\Delta h$, and since the residual term represents a flow rate, the relaxation coefficient must be an impedance. Hence the relaxation coefficient may be regarded as the equivalent impedance of the polygonal sides joining a node
to its neighbours. Thus,

$$
\begin{equation*}
\operatorname{RELAX}_{B}=\frac{1}{\sum_{i} Y_{i, B}+\frac{A_{B} S_{B}}{\Delta t}} \cdots \tag{13}
\end{equation*}
$$

The above method for the simultaneous solution of Eq.11 is essentially that of Gauss-Seidel and therefore unconditionally convergent.

It should be noted that the value of 1 of the nominator in Eq. 13 is in fact a coefficient whose value ranges between 0.8 and 1.2 . It can be varied to speed up convergence. This does nor affect the end results. The optimal value of this coefficient can be obtained by making a number of test runs with actual data and comparing programe run times. In this study we used a coefficient of 0.8 . The choice of a value for ERROR is important for two reasons:

1) it determines the accuracy of the final results, and
2) it is a factor in determining the amount of machine time necessary for the relaxation process and therefore directly influences: the cost of the job.

Only a few months' data need to be run to find out what value of ERROR can best be used. A suitable value of 5 million $\mathrm{m}^{3}$ a year was found.

Finally there is the problem of choosing a proper value of $\Delta t$, the time step in years to be used. Its choice is initially arbitrary, although if chosen too big the approximation of finite differences to differentials will cease to be valid and, assuming that the process will converge, the results will be in error. The programme sets a maximum value of one month ( $1 / 12$ th year). For a set of actual data the prograrme is run, using different values of $\Delta t$. The results are compared and the maximum $\Delta t$. For which the results do not change appreciably is determined. In this study we used $\Delta t=18$ days or $1 / 20$ th year.

A simplified flow chart for the digital computer solution is shown in Fig. 17 and a portion of the Fortran programe in Fig. 18.



Fig.18. Part of computer progronme.


### 3.7 Varamin groundwater simulation model

The simulation model of the Varamin groundwater basin was developed in three stages: an asymmetric network was constructed, basin parameters and input data were prepared, and finally the model was verified. These three stages will be explained in more detail below.

### 3.7.1 Construction of nodal network

Prior to the construction of the basin's nodal network, a careful study of its geohydrology was made. Fortunately a considerable amount of information had been collected in previous years,including data on soils, climate, subsurface geology, topography, aquifers and aquifer characteristics, monthly water table elevations over a four-year period, groundwater quality, surface water and its distribution over the Plain in various periods of the year, water rights of villages, cultivated and non-cultivated areas, groundwater excractions by wells and qanats in various parts of the Plain, cropping patterns, consumptive use of various crops grown in the Plain, and irrigation practices.

All this information in map form was used to decide the location of the natural and arbitrary boundaries of the basin and the size, number, and distribution of the polygons to be constructed.

The polygonal network that was eventually accepted is shown in Fig. 19. As can be seen from this figure, impervious boundaries were assumed at the north and south where the Elburz Range and Siah Kuh Range limit the basin. These mountain ranges are composed of indurated Bakhtiary formations and Miocene marls, which do not transmit groundwater in substantial quantities. Similarly, the protruding Pishva Hills act as an impervious boundary, being composed of the same type of rocks. At the northwest the basin boundary coincides with the groundwater divide. At the west the boundary is formed by the outcropping impervious Miocene basement. The main branch of the Jaj Rud roughly follows this boundary. This river branch is dry for most of the year. In spring it acts as a spillway during floods of short duration.

Arbitrary boundaries occur at the northeast and southeast; they are open boundaries through which certain, though small, amounts of groundwater flow out of the basin. These boundaries were, however, assumed to be closed and the subsurface


Fig.19. Asymmetric network of the Varcomin groundwater basin.
outflows through them were accounted for in the assigned external flow or AQvalue of the polygons in question as though they were pumped extractions.

Two further boundary problems merit attention. Firstly, the impervious boundary at the north, represented by the Elburz Range, is indeed closed except for the relatively wide Jaj Rud valley mouth. Borings have revealed that the river has deposited very coarse materials on the valley floor. The thickness of these sediments increases in downstream direction and at the mouth may be more than 50 m . Considerable quantities of river water are lost in these sediments and they enter the basin (Polygon I) as a subsurface inflow. The rate of inflow was estimated and accounted for in the $A Q-v a l u e$ of this polygon as though it were a recharge.

The second boundary problem we faced was at the southern limits of the basin, more specifically where the Jaj Rud leaves the basin. We had to know whecher there is any subsurface outflow at this site. Geo-electrical soundings across the Jaj Rud outlet showed that no thick layers of coarse materials occur here as they do in the north, and shallow boreholes revealed clayey materials on Miocene marls.Hence we felt it safe to conclude that no substantial outflow of groundwater occurs at the outlet, a conclusion which is corroborated by the water table contour map (Fig.20).

As to the number and size of the polygons, we realized that there were not sufficient basic data available for a fine network,besides which a fine net-work would increase the computer time and the time required for data preparation.

A glance at the water table contour map (Fig. 20) shows a rather abrupt increase in wacer cable gradient in the middle of the Plain. This is caused by the protruding Pishva Hills, which can be traced underground in north-westerly direction. These hills are partly eroded and breached by the Jaj Rud. This geologic feature causes a contraction of the groundwater flow, as a result of which the water table gradient increases in this zone. In contrast, rather flat water table gradients occur in the northern and far southern parts of the Plain. The latter area is a true salt desert without any agricultural activity.

These differences in water table gradient made it necessary to vary the size of the polygons to obtain the same accuracy throughout. Hence in the middle of the Plain the polygons are smaller than in the northern and southern parts. The polygons in the middle vary from about 2,000 to 3,000 ha and in the other parts from 4,000 to 10,000 ha. The two southern-most polygons, located in the salt desert, are roughly 15,000 and 17,000 ha.


Fig. 20. Water tabie contour map (average for 1966 ).

Some 90 water table observation wells exist in the Plain and, although it is common practice to choose an appropriate number of such wells to construct a network,most of them could not be used because of their unsuitable siting. Hence a set of 27 node points were selected in such a manner that small polygons were obtained in the middle of the Plain and large ones in the other parts. The network of polygons was constructed by applying the Thiessen method,i.e. the node points were connected by lines which formed tiriangles, and perpendicular bisectors to the interconnects were constructed to form the boundaries of the polygons. It should be noted that, to avoid computational errors, the interior angles of the triangles had to be less than $90^{\circ}$ to ensure that no interior angle of the polygons was less than $90^{\circ}$.

Finally, consecutive numbers were assigned to all the node points (Nos. 1 to 27) and polygon sides (Nos. 1 to 61).

### 3.7.2 Preparation of data

Once the network of polygons had been constructed, the next step was to prepare various geological and hydrological data. A brief description of this data preparation follows.

Transmissivity

Transmissivity is the ability of an aquifer to transmit water and is defined as $T=K D$,or the product of the saturated thickness of the aquifer and the average hydraulic conductivity for horizontal groundwater flow ( $T$ is expressed in $\mathrm{m}^{2}$ /day). The most reliable information on aquifer transmissivity is obtained from pumping test data.

In the past, pumping tests had been performed on 44 deep wells, rather regularly distributed over the Plain. One test had even been conducted on an exploratory well near hesar Goli in the south-eastern part of the desert area. Most of these tests were simple well tests and only a few had been conducted on a well that had a number of piezometers in its vicinity. The data of the drawdown observed in the wells and/or piezometers were used to calculate the transmissivity of the aquifer.

It is obvious that these, 44 transmisaivity data, though most useful, were too few for our purpose. Therefore we gathered additional information on the magnitude


Fig.21. Transmissivity map.
and variation of aquifer transmissivity from the results of geological investigations and existing water wells. Drillers' logs of water wells,located in different parts of the Plain, were collected and analyzed to delineate the waterproducing zones. We assigned hydraulic conductivity values to the various sand layers and calculated the transmissivity for each layer. The sum of the values yielded the transmissivity of the aquifer. Corrections were made for partial penetration. In this way we estimated the aquifer transmissivity at 160 different sites in the Plain. All these data were used to prepare a transmissivity map, which is shown in Fig. 21.

The transmissivity values for each polygonal side chen had to be found. To do this we superimposed the transmissivity map on that showing the network of polygons and calculated a weighted average transmissivity of each polygonal side.

Finally we measured on the map the length of the interconnect $L$ and that of the polygon side $W$. In accordance with Eq. 10 , we then calculated the conductance factor $Y$ by multiplying the values of $W$ and $T$ and dividing the result by the value of $L$.

Water table changes affect the value of the conductance factor: a rise in water table causes the transmissivity to increase and a drop causes it to decrease. Hence the value of $Y$ has to be corrected if. such changes occur. For each time step the computer corrected the value of $Y$ by multiplying it with the ratio: saturated thickness of the aquifer/total thickness of the aquifer.

Since many of the transmissivity vatues were only rough estimates, they were subject to error. Such errors were checked later during the calibration process and corrected.

## Storage coefficient

The storage factor AS, which is the product of the area $A$ of a polygon and the average storage coefficient $S$ of that polygon's water-bearing sediments, is considered to be a measure of the storage characteristics of each polygon.

Because of lack of time and funds, it was not possible for us to make field determinations of the storage coefficient or specific yield. Hence we combined ail available geologic data and our professional judgement to prepare representa-


Pig.22. Storage coefficient map.
tive storage coefficients for each polygon. Storage coefficients vary as the lithology of the water-bearing layers varies. Differences can also be found in the same material for a rising and a falling water table. The computer programe used in this study assumed that the storage coefficient did not change as water tables changed. This was justified, as tests on models have shown that the differtree in water table responses between uniform and non-uniform storage factors is small enough to be neglected. The logs of 165 deep wells scattered over the Plain were examined and the materials described were grouped into six major lithological groups. A storage coefficient was assigned to each group,e.g. 25 per cent was assigned to the "gravel" group, which comprised all coarse materials,and 2 per cent to the "clay" group, which comprised all heavy textured materials, (heavy clay, tight clay, sticky clay, etc.). A weighted storage coefficient was calculated for each well site, from the land surface to about 10 m below the water table: bed thickness was multiplied by the assigned storage coefficient, the results were added, and divided by the total thickness of the examined profile.

The storage coefficients thus found were plotted on a map and lines of equal storage coefficient drawn (Fig.22). In the head of the alluvial fan, where coarse materials predominate, the storage coefficient varied from 15 to 20 per cent. In domstream direction it decreased and assumed values of less than 5 per cent in the finer textured materials of the border zones.

The polygonal network map was superimposed on the storage coefficient map and an average weighted storage coefficient was calculated for each polygon. Multiplying the value by that of the polygonal area yielded the storage factor AS.

Water table data

Since historic water tables at one single well site usually do not represent water cable elevations in an entire polygon and, moreover, since the arbitrary node points we chose did not coincide with a water table observation well, well hydrographs and water table contour maps were used to find representative water table elevations for each polygon. Such maps are the only possible tools if actual records are missing or are of poor quality.

Historic records (monthly observations) from about 90 observation wells were available, covering a period of nearly four years. Monthly water table contour maps were drawn, an example of which is shown in Fig. 20. The historic records
used started in October 1963. The water table contour map of this date was superimposed on the polygon network map and representative initial water table elevations (HZERO) were assigned for all the 27 polygons.Representative quarteryear water table elevations for all 27 polygons were thus determined for the remaining historic period.

During the calibration process, when the validity of the model was tested, nodal water table elevations generated by the computer were compared with these historic water table elevations.

Overall groundwater balarce

The basin's overall hydrologic balance was assessed, after which the specific recharge and discharge components of each polygon were estimated. A brief discussion of the basin's hydrologic balance follows.

## Rainfall

LLOYD, DRENNAN, and BENNELL (1966) found that direct recharge does not occur in areas receiving less than 220 mm of rainfall and that above this value the amount of recharge depends on the total rainfall and the way it is distributed. Recharge studies in California (DAVIS and DE WIEST, 1966) have shown that no recharge takes place when the rainfall is less than 127 to 254 mo ( 5 to 10 inches), except on very permeable soils.

From Table 4 it can be seen that the average rainfall on the Plain is 141 mm per year. Hence rainfall cannot be regarded as an important source of groundwater recharge. Some recharge may occur if heavy rain falls on land that has just been irrigated, but since rain rarely falls during the irrigation season, it does not contribute more chan a few million $\mathrm{m}^{3}$ to the groundwater recharge.

## Canal and irrigation percolation

The two main recharge components are the seepage losses in the system of irrigation canals and distributaries and the percolation losses in the fieids. These components are difficult co estimate. In the Plain there are 125 villages that have water rights and the available river water is distributed in accordance with these rights. In spring, when the discharge of the river is high, all villages receive
their portion, but in autumn, when the discharge is low, water is only supplied to those villages that do not posses a well or qanat. Since no engineering works exist and the water is distributed by labourers working with shovels, the precise amount of river water delivered to each village is not known.

The length of the irrigation canals and distributaries was measured from the topographical map and found to be 1405 km .

In the hydrological year 1965/66, when approximately 254 million $\mathrm{m}^{3}$ of river water was available to the Plain, canal seepage measurements were taken in nine polygons (Nos.1, 2, 5, 6, 8, $11,20,21$, and 25).Furthermore, it was assumed that the canals flowed half full for 100 days of the year. The total seepage loss from the canal system was thus found to be $50 \mathrm{million} \mathrm{m}^{3}$. The seepage losses - of the other historical years were calculated as a proportion of the respective river discharges in those years.

Besides canals and distributaries, there are also well and ganat ditches which lose water to the underground. The total length of these ditches is approximately 300 km . Their seepage was calculated by analogy to the irrigation canal seepage. It was found to be of the order of 13 million $\mathrm{m}^{3}$. For the other historical years this seepage was calculated as a proportion of the annual well and qanat extractions.

In quantifying the percolation losses in the fields, we encountered major difficulties because no land use maps nor information on cropping patterns were available. Whatever information we could collect on land use, cropping patterns, villages' water rights, well and qanat discharges, consumptive use of the crops grown, and hectares of land under crops was used to estimate the total net percolation in the field. It was found to be some 125 million $\mathrm{m}^{3}$ a year.

The total annual groundwater recharge rom canal and ditch seepage and irrigation percolation was therefore $13+50+125=j 88$ miliion $\mathrm{m}^{3}$.

## Welt and qanat discharges

Since 1963, inventories have been kept of the number of qanats and shallow and deep wells existing in the Plain. Their yields were measured by Parshai flume (qanat yields were measured seasonally). Because the wells were not equipped with meters and no records of pumping were kept, inquiries were made to each
individual owner as to the hours of operation per day, the number of days per monch, and the number of months per year. On the basis of these data the total annual well and qanat extractions were calculated. The results are shown in Table 7.

## Euaporation

The lowermost part of the Plain has fine-textured soils with a shallow water table, ranging from less than 1 m to 5 m below the land surface. Inder the prevailing climatological conditions, these factors are conducive to severe salinization of the soil by capillary rise from the groundwater, which is very salty. Since this part of the Plain is a true salt desert, there is clear evidence of such capillary rise.

Because the type of soil occurring in this area (3],582 ha) is not precisely known, the total quantity of water lost by capillary rise from the ground water cannot be determined. For two types of soil, fine sandy loam and silty clay loam, the capillary rise was calculated, using the equation (RIJTEMA, 1965)

$$
\begin{equation*}
z=\frac{k_{0} \psi}{v+k_{o}} \quad 0<\psi \leqslant \psi_{a} \tag{13}
\end{equation*}
$$

where
$z=$ depth to the water table (cm)
$k_{o}=$ capillary conductivity at $\psi=0(\mathrm{~cm} /$ day)
$\psi=$ suction of soil moisture (cm)
$v=$ flow velocity of water in the soil (cm/day)
$\psi_{a}=$ suction at the air entry point (cm)

The suction range in which the function $k=k_{o} e^{-\alpha\left(\psi-\psi_{\max }\right)}$ holds gives the following equation

$$
\begin{equation*}
z=1 / \alpha \ln \frac{v+k_{o}}{v+k_{o} e^{-\alpha\left(\psi-\psi_{a}\right)}}+\frac{k_{o} \psi_{a}}{v+k_{o}} \quad \psi_{a} \leqslant \psi \leqslant \psi_{\max } \tag{14}
\end{equation*}
$$

WIND (1955) introduced the equation

$$
\begin{equation*}
z=2 / 3\left(\frac{a}{v}\right)^{2 / 3}\left[\frac{1}{2} \ln \left(x^{2}-x+1\right)+\sqrt{3} \arctan \frac{2 x-1}{\sqrt{3}}-\ln (x+1)\right]+c \tag{15}
\end{equation*}
$$

where

$$
\begin{aligned}
& x=\left(\frac{v}{a}\right)^{1 / 3} \psi^{1 / 2} \\
& c=\text { integration constant }
\end{aligned}
$$

The values of the different parameters in these equations are, for fine sandy 1oam:

$$
\begin{array}{ll}
k_{0}=12 \mathrm{~cm} / \mathrm{day} & \alpha=0.0248 \mathrm{~cm}^{-1} \quad a=12.0 \mathrm{~cm}^{2.5} \mathrm{day}^{-1} \\
\psi_{a}=10 \mathrm{~cm} & \psi_{\max }=300 \mathrm{~cm}
\end{array}
$$

and, for silty clay loam:

$$
\begin{array}{lll}
k_{o}=1.5 \mathrm{~cm} / \text { day } & \alpha=0.0237 \mathrm{~cm}^{-1} & a=20 \mathrm{~cm}^{2.5} \mathrm{day}^{-1} \\
\Psi_{\mathrm{a}}=0 & \psi_{\max }=300 \mathrm{~cm} &
\end{array}
$$

The constant $C$ in Eq. 15 can be calculated by applying Eq. 14 for various values of v . Eqs.13, 14 , and 15 are then applied to find the relationship between $\psi$ and $z$ for various values'of $v$.

For the above two soil types Figure 23 shows the relation between the maximum capillary rise for an assumed maximum suction of $15,000 \mathrm{~cm}$ and the depth to the groundwater table. These graphs refer to homogeneous soils. In non-homogeneous soil profiles, in which the soil gradually changes from a coarse sandy material at the water table to a fine cextured material at the land surface, the capillary rise can still be considerable, even with a deep groundwater table. Under such conditions the favourable capillary properties of a coarse textured soil are present in the, wet range, whereas the better capillary properties of a finer textured soil become dominant in the range of high suction.


Fig.23. Relation between the maximum capillayy mise, $V$, for a suction of 15,000 om and the depth of the water tabie, $Z$, for two different types of soil.

The depth to the water table was known in 18 observation wells in the southern part of the Plain (Fig.24), By making use of Fig. 23 , the maximum capillary rise from the groundwater at these sites could be found. Lines of equal capillary rise were drawn and the acreages of equal capillary rise were measured on the map. Assuming that the entire area was made. up of fine sandy loam, we calculated the possible maximum evaporation from the area. to be of the order of 65 million $\mathrm{m}^{3}$. If the entire area were made up of silty clay loan, the maximum possible evaporation would be of the order of 45 million $m^{3}$ a year.

Although little is known about the soil types occurring in these desert areas, some well logs suggest the presence of a soil type heavier than a.fine sandy loam. Self-mulching and the presence of salt crust on the land surface may have a reducing effect on the evaporation rate. Atmospheric conditions, in particular during winter, may also restrict the evaporation rate. Hence the actual quantity of water lost from this area by capillary rise and evaporation may be less chan 45 million $\mathrm{m}^{3}$ a year. But any value between 0 and 45 million $\mathrm{m}^{3}$


Fig. 84. Depth to the water table.
which gives a closed balance can be explained by evaporation from this area.

Change in storage
Owing to the below-average river discharge and the high rate of groundwater abstraction during the historical years considered, the water table declined. This fall varied from a few centimeters to more than 1 m , depending on the abstraction rate and the well pattern. Areas with an equal fall were delineated on the map and, using the assigned storage coefficients, we calculated the quantity of groundwater that was lost from storage annually. We found it to average out at some 25 million $m^{3}$ a year.

## Groundwater balance

The above approximate information allowed the following groundwater balance to be assessed for the basin (average for the period 1963/64-1966/67).

| Subsurface inflow | $65 \times 10^{6} \mathrm{~mm}^{3}$ | Subsurface ourflow | $5 \times 10^{6} \mathrm{~m}^{3}$ |
| :---: | :---: | :---: | :---: |
| Seepage from canals | 50 | Well and qanat abstraction | 245 |
| Seepage from well ditches | 13 | Evaporation in desert | 33 |
| Field percolation | 125 | Change in storage | - 25 |
| Percolating rain and runoff from hill creeks | 5 | - |  |
| total | $258 \times 10^{6} \mathrm{~m}^{3}$ |  | $258 \times 10^{6} \mathrm{~m}^{3}$ |

The average river flow at Darvazeh for the period considered was $263 \times 10^{6} \mathrm{~m}^{3}$ a year. If we assume that an average of $18 \times 10^{6} \mathrm{~m}^{3}$ a year was lost by deep percolation in the gravel cract downstream of this site, an average of $245 \times 10^{6} \mathrm{~m}^{3}$ river water was annually available for irrigation in the Plain. The average abstraction from wells and qanats for the period considered was of the same magnitude: $245 \times 10^{6} \mathrm{~m}^{3}$ a year. Hence a total volume of $490 \times 10^{6} \mathrm{~m}^{3}$ a year was available for irrigation.

Since the percolation losses from the irrigation canals and well ditches were of the order of $63 \times 10^{6} \mathrm{~m}^{3}$ a year, the total quantity of irrigation water available to the farms was $490 \times 10^{6} \mathrm{~m}^{3}$. minus $69 \times 10^{6} \mathrm{~m}^{3}=427 \times 10^{6} \mathrm{~m}^{3}$ a year.

The four-year period used to assess this groundwater balance was a rather dry one (average river flow only $263 \times 10^{6} \mathrm{~m}^{3}$, whereas for the entire 22 -year period the average was $364 \times 10^{6} \mathrm{~m}^{3}$ a year). Although precise historical data on the irrigated acreage do not exist, project experience shows that it could not have been much more than an average of $21,000 \mathrm{ha}$. With an average consumptive use of $7,450 \mathrm{~m}^{3} / \mathrm{ha}$, this means that the total quantity of water used consumptively by the crops was of the order of $156 \times 10^{6} \mathrm{~m}^{3}$ a year, corresponding with an overall efficiency of 32 per cent.

From the data on the total quantity of irrigation water available to the farms and the total quantity of water used consumptively, we calculated an average total water loss on the farms of $427 \times 10^{6} \mathrm{~m}^{3}$ minus $156 \times 10^{6} \mathrm{~m}^{3}=271 \times 10^{6} \mathrm{~m}^{3}$ a year. Not all this water percolated to the water table; deep percolation was assumed to be of the order of $125 \times 10^{6} \mathrm{~m}^{3}$ a year, the remaining portion being lost through surface runoff and evapo(transpi)ration. Figure 25 shows a schematic of the hydrologic system of the Varamin basin.


Fig.25. Schematic of hydrologic system of the Varamin basin (quantities in million $\mathrm{m}^{3}$ ).

Polygonal net deep percolation

Once an overall hydrologic balance of the basin had been assessed, an estimate of the historic net recharge or discharge for each polygonal area had to be made. This was the most difficult of the data to prepare because of the lack of reliable historic data on cultivated land, cropping patterns, surface water and groundwater supplied, etc.

The most complete data were available for the year $1965 / 66$ and these data were used to calculate the net deep percolation for each polygon. The polygon network map was superimposed on the maps showing the villages with their water rights, the canal system, the location of the deep and shallow wells and ganats and their discharges. The length of the canals and ditches in each polygon was measured on the map and the total quantity of river water supplied to each polygon was estimated. The total groundwater extraction by wells and qanats in each polygon was calculated. Knowing the acreage of land under cultivation and the crops grown in the polygons, we could estimate for each polygon the total recharge from canal and ditch seepage and field percolation. From these recharge figures we subtracted the total groundwater extraction for each polygon and obtained their net recharge, also called net deep percolation or AQ value.

For the other historical years, we calculated the net deep polygonal percolation as a proportion of the available river water and groundwater during those years. The four-year net deep percolation values were fed into the computer and, using the sloping line method (THOMAS, 1973), we obtained the polygonal AQ values for intermediate times.

Subsurface inflow or outflow through open boundaries was accounted for in the $A Q$ values of the respective polygons:

> in Polygon 1 the AQ value was enlaxged by 65 million $m^{3}$ (river underflow); in Polygon 3 it was diminished by 2 million $m^{3}$, and in Polygon 27 by 3 million $m^{3}$ (subsurface outflow).

## Ground surface elevation

Among the other data prepared for the groundwater model was the elevation of the ground surface for each node point, denoted as SL. For this purpose the
polygon network map was superimposed on a topographic map with contour lines of the ground surface and the elevation for each of the 27 node points was read or found by interpolation between contour lines.

Elevation of the impervious base

The model also required data on the elevation of the impervious base at each of the 27 node points, denoted as BL. We took as this base the Miocene/Pliocene deposits, which are predominantly marl and clay.

A contour map of the surface of these deposits was drawn, using data from deep wells and those of Figs. 3 and 4, We then superimposed the polygon network map on this map and determined the elevation of the impervious base for each node point.

Elevation of the impervious base at the mid-point of the flow path

The model further required the elevation of the impervious base at the mid-point of the flow path, i.e. at the 61 branches or polygon sides. The same procedure as described above was followed to find these elevations, which were denoted as BCK.

## Elevation of the arainage base

If the water table is rising, it may eventually reach the bottom of a drain, or if there are no drains, it may reach the land surface, after which no further rise is possible.Any groundwater entering the drain or reaching the land surface is accounted for as surface water flow. Hence the model required an upper limit to which the water table can rise; this was denoted as HS. The elevation of the polygonal drainage base (HS values) was assumed to equal the elevation of the ground surface, as discussed earlier.

### 3.7.3 Calibration of the model

An examination of the computer output showed that for most polygons the water table elevations generated by the computer matched the historical water table elevations reasonably well. There were, however, some deviations, caused by errors in the values of the storage coefficients, transmissivities, or net deep percolations. Calibrating the model consisted of correcting these errors to obtain a closer match of the water table elevations.

Further corrections and adjustments were made and several new test runs were done on the computer until the computed water table elevations for all nodes matched (closely enough) the historic water rable elevations (Fig. 26).


Fig. 26. Flow chart of model verification.

## 4. Agricultural planning

Any attempt to prepare a detailed agricultural production programme ought to be based on carefully analysed and verified data on inputs applied and yields obtained, which should be representative of farm types, soil properties, agricultural seasons, and other ecological conditions.

These data were not available to the planning team, as farmers were illiterate and did not keep daily account of their transactions. Data were particularly lacking on the vital subject of water, because the measurement of water applied to crops is technically complicated and was thus beyond the farmers' ability.

A Farm Management Survey of 51 farms in the Varamin Plain (EREZ, 1967), although only based on the farmers' memory, provided valuable information about land use, cropping patterns, sales of produce, farm expenses, change of stock, and similar matters. It was, however, inadequate for the evaluation of input levels and returns. Another attempt to gather crop budget data was made as part of a Socio-Economic Survey of 1,400 farmers in the Plain, but was equally abortive from the viewpoint of collecting refinable input data (EREZ and BAHODORI, 1967).

Even if the surveys had produced reliable analytical data, they would have to have been adapted to new cropping techniques, new varieties, new crops, improved irrigation and land preparation techniques, and similat innovations of modern technology. Such data can only be derived from experimental plots and field tests, activities which the planning team commenced only in the last stage of the project. Since a minimum of 3 years of verification and replication of such tests is required, no sound basis for agricultural planning existed.

The problem itself therefore represented a classical decision-making problem under conditions of uncertainty and risk. This situation called for careful efforts to define priorities and for the application of sensitivity tests to evaluate the range and possible impact of the uncertainties in the available or assumed data and in the input-output coefficients. The planners' dilema between the lack of reliable data and the need, to make planning proposals became a challenge for refining and testing the already available information and for the development of planning techniques that could overcome the problems.

In line with this approach, linear programming was applied in attempts to plan the future agricultural production patterns and to test the sensitivity of the various assumed data by more refined tests of the impact of their uncertainty on the optimal solution. This was probably the first attempt to use such a technique for agricultural planning in Iran.

### 4.1 The model

Based on representative findings from the Farm Management Survey and on the results of field tests from the Centrat Experimentation Station of Varamin, a model farm was designed, having the following resources:

Land: 13 ha of Class I soils

Labour: monthly labour capacities, varying between 65 and 45 man days,representing an average farmer family with two sons (16 and 10) and one daughter (12), and taking into consideration the observance of Islamic and national holidays

Water: based on the then prevailing water rights and the water supply system in the Varamin Plain, the following river and qanat water supplies were assumed for each month (Table 8):

| TABLE 8. Surface water and qanat water supplie's |  |  |
| :--- | :---: | :---: |
| Manth | - | \% of total |
| October | - | - |
| November | - | - |
| December | - | - |
| January | 900 | - |
| February | 3,850 | 3.3 |
| March | 9,050 | 14.2 |
| April | 7,600 | 33.5 |
| May | 4,100 | 28.1 |
| June | 1,550 | 15.2 |
| July | - | 5.7 |
| August | - | - |
| Septenber | 27,050 | - |
| TOTAL |  | 100.0 |

This water supply pattern represents not only the prevailing water rights of a Varamin farmer, but also the monthly discharge figures of the Jaj Rud (Table 6)

Capital: A revolving credit of 5,000 Rials from the village cooperative was assumed, representing the constraints of this source. Additional own sources were not taken into consideration, being negligible. There are no fixed production assets like tractors and machinery on the farm; their services are hired, as is usual for most farms in the area.

The model farm includes a range of production activities, as outlined in Table 9. Other production activities on the farm are the hiring of labour, the purchase of water, the borrowing of capital, and the provision of feeding alternatives.

Generally, the linear programming model was defined as follows:

$$
\operatorname{Max} . C=\sum_{j=1}^{n} c_{j} x_{j}
$$

subject to

$$
\begin{aligned}
& \sum_{j=1}^{n} a_{i j} x_{j}\left\{\geqslant \underset{i}{\leqslant} b_{i} \quad \text { for } i=1 \ldots\right. \text { m } \\
& x_{j}>0
\end{aligned}
$$

where
$c_{j}$ represents net returns (or costs) of the various activities
$a_{i j}$ represents the various coefficients
The activities $x_{j}$. include the dairy and field crop production activities, the feeding activities, and the various monthly supply alternatives of hired labour, purchased water, and borrowed capital. The constraints $b_{i}$ include the available land,monthly water supplies of river and/or ganat water, family labour and several "agro-technical limitations, such as considerations of crop rotations, composition of various fodder sources, and others.

TABLE 9. Production activities for the model farm

| Number |  | Acrivity |  | Yield |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Silage traize | - June ${ }^{1}$ | 2,400 | F.U. ${ }^{2}$ |
| 2 | Silage maize | - July | 2,800 | F.U. |
| 3 | Silage maize | - August | 3,200 | F.U. |
| 4 | Silage maize | - September | 3,200 | F.U. |
| 5 | Silage maize | - October | 3,200 | F.U. |
| 6 | Silage maize | - Novenber | 2,800 | F.U. |
| 7 | Fodder maize | - June | 3,000 | F.U. |
| 8 | Fodder maize | - July | 3,500 | F.0. |
| 9 | Fodder maize | - August | 4,000 | F.U. |
| 10 | Fodder maize | - September | 4,000 | F.U. |
| 11 | Fodder maize | - October | 4,000 | F.U. |
| 12 | Fodder maize | - November | 3,500 | F. ${ }^{\text {P. }}$ |
| 13 | Oats | April | 2,000 | F.U. |
| 14 | Oats | - May | 2,500 | F.t. |
| 15 | Alfalfa hay | - Selling | 12,000 | $\mathrm{kg} / \mathrm{ha}$ |
| 16 | Alfalfa hay | - Feeding | 12,000 | kg/ha |
| 17 | Cotton |  | 2,500 | $\mathrm{kg} / \mathrm{ha}$ |
| 18 | Tomato | - Spring | . 30,000 | $\mathrm{kg} / \mathrm{ha}$ |
| 19 | Cucumber | - Spring | 17,000 | kg/ha |
| 20 | Wheat |  | 3,000 | $\mathrm{kg} / \mathrm{ha}$ |
| 21 | Eggplant |  | 25,000 | kg/ha |
| 22 | Sugar-melon | - Spring | 23,000 | kg/ha |
| 23 | Squash | - Spring | 23,000 | kg/ha |
| 24 | Cucumber | - Fall | 13,000 | kg/ha |
| 25 | Sorghum |  | 4,000 | kg/ha |
| 26 | Maize grain |  | 4,000 | kg/ha |
| 27 | Alfalfa green | fodder | 5,000 | F.u. |
| 28 | Dairy cows |  | 2,300 | 1/year |

[^0]This linear programming model had a.matrix of 165 columns and 103 rows. It was utilized for the following sensitivity tests:
a) Impact of increasing interest rates on borrowed capital
b) Impact of varied (uncertain) water requirement coefficients
c) Impact of an assumed future water supply policy

For the tests under a) the interest rates were varied from 6\%, as included in the initial master matrix, to $10 \%, 15 \%$ and $20 \%$. For those under b) the used water requirement coefficients were each time increased and decreased by $20 \%$. For the tests under c) four different water supply alternatives were assumed, representing various steps towards a fully integrated new water supply system in the Varamin Plain.

These parametric runs measured the impact of an assumed water supply function and tested the effect of the supply of larger, but costlier, water quantities on the optimal production plan and on the obtainable farm net income. At the time this model was developed, we could not define a more precise water supply function, because data on proposed dam sites, diversion systems, recharge basins, additional wells, canal conveyance systems, etc., were not available.

Another aspect was the evaluation of the impact of seasonally adjusted supply and pricing policies, which were also still undetermined but which could have a decisive impact on optimal agricultural production plans. Lacking such basic daṭa we used a set of dummy data for the definition of the upper and the lower limits of a future water supply function. Thus, the calculated values of marginal product (shadow prices) of the various seasonal and annual restrictions could indicate not only a required optimal monthly supply curve for agricultural pro* duction and the optimal cropping patterns, but also the economic implications of investments in seasonal supply regulations, according to estimated peak season demands. These considetations were defined by the four water supply alternatives shown in Table 10 , which are all based on an imaginary water supply function curve.

The model did not take any market constraints into consideration. The cash erops (wheat, cotton and sumer grains) were being sold at controlled guaranteed prices. With regard to alfalfa hay, the supply could - at the time of our study - only partially meet the demand and the production of this cash crop is constantly
expanding. As to the vegetable crops, they were being supplied to the nearby market of Teheran. The market situation for vegetables was still being investigated during our study,taking into account the competitive supplies from other production areas and the expected output of other new irrigation projects in the southern and western parts of Lran, which were also intended to supply the Teheran market.

TABLE 10. Four assumed water supply alternatives for a model farm

## Alternattives and assumptions

Alternative 1. River and sanat water as outlined in Table B; their price is assumed zero. Well water, which can be purchased by che farmer, is restricted to a maximum monthly quantity of $1,700 \mathrm{~m}^{3} / \mathrm{ha}$ or $22,000 \mathrm{~m}^{3}$ for the whole farm of 13 ha ; in addition there is an annual constraint of well water buying of $9,000 \mathrm{~m}^{3} /$ ha and therefore $117,000 \mathrm{~m}^{3}$ for the thole farm. The price of purchased well water is $0.4 \mathrm{Rls} / \mathrm{m}^{3}$. These conditions and restrictions represent the water bupply situation in the Varamin Plain at the time of our study.

Altermative 2. River and qanat water as in Alternative 1 . Well water buying is further restricted; $20 \%$ less water than in Alternative 1 ; during the months of April to November the farm will be permitted to buy a maxioum amount of $17,600 \mathrm{~m}^{3}$ only; during the rest of the year, when water deasnd is low, the restriction of $22,000 \mathrm{~m}^{3}$ will prevail. In addition to the buying constraints during the main irrigation season, there is an annual limitation on the purchase of well water, amounting to goz of the similar constraint in Alternative 1 .
The price of well water is unchanged: $0.4 \mathrm{Rls} / \mathrm{m}^{3}$. This alternative represents $a$ possible reaction by the supply authorities to the withdrawal of part of the Jaj Rud water for urban consumption in Teheran. It will be necessary to introduce well water allocations and farmers will not be petmitted to buy more chan a certain maximum per ha and per farm. No higher well water prices are taken into consideration, as the authorities would not be able to restrict che cotal water supply and simultaneously increase the water prices.

Altemative 3 . River and ganat water as in Alternative 1 . Well water constraints are released: 257 more water is available than in Alternative 1 . Thus the maximum monthly quantity now amounts to $27,500 \mathrm{~m}^{3}$, and the maximum annual quantity to $146,250 \cdot \mathrm{~m}^{3}$.
The monthly well water prices are differentiated and represent the monthly demand patterns; thus during the main irrigation season they vary from $1.10 \mathrm{kls} / \mathrm{m}^{3}$ to $1.35 \mathrm{Ris} / \mathrm{m}^{3}$, and during the rest of the year they are assumed constant at $1.0 \mathrm{Rls} / \mathrm{m}^{3}$.
This alternatíve represents a combination of basic water rights for river and qanat water, with the eatablishment of a market for well water, which will induce the construction of more wells.

Alternative 4. A full integration of the water supply system has been achieved. Compared with Alternative 1 , the maximam monthly constraints are increased by $75 \%$, amounting . to $38,500 \mathrm{~m}^{3}$ per month and to $204,850 \mathrm{~m}^{3}$ per year for the whole farm.
to $38,500 \mathrm{~m}^{2}$ per month and to $204, B 50 \mathrm{~m}^{2}$ per year for the whole farm.
Water prices during the peak season (April no November) vary from $2.2 \mathrm{Rls} / \mathrm{m}^{3}$ to $2.7 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$ and during the rest of the year are constant at $2.0 \mathrm{Rls} / \mathrm{m}^{3}$. This alternative represents a water allocation policy of a modern integrated supply system, which sella water at cost, without any subsidies.

The following monthly quantities and prices were assumed for each one of the four alternatives (Table 11).

TABLE 11. Monthly guantities of river and well water and well water prices for the four alternatives

| Month | Altermative. 7 |  |  |  | Altermative |  |  | 2 | Attermative |  |  | 3 | Altermative 4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Q River } \\ \mathrm{m}^{3} \end{gathered}$ | $\begin{gathered} \text { Q We1t } \\ \mathrm{m}^{3} \end{gathered}$ | $\begin{aligned} & \text { P Well } \\ & \text { Rls } / \mathrm{m}^{3} \end{aligned}$ |  |  | River $\mathrm{m}^{3}$ | $\begin{gathered} \text { Q Wel1 } \\ \mathbf{m}^{3} \end{gathered}$ | P Weil $\mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$ |  | River $\mathrm{ma}^{3}$ | $\begin{gathered} \text { Q Well } \\ \mathrm{m}^{3} \end{gathered}$ | $\begin{aligned} & \text { P Well } \\ & \text { R1s/on } \end{aligned}$ | $\mathrm{Q}^{\mathrm{a}}$ | P <br> $\mathrm{R1s} / \mathrm{m}^{3}$ |
| Sept. | - | 22,000 | 0.4 |  |  | - | 17,600 | 0.4 |  | - | 27,500 | 1.30 | 38,500 | 2.60 |
| Oct. | - | 22,000 | 0.4 |  |  | - | 17,600 | 0.4 |  | - | 27,500 | 4.20 | 38,500 | 2.40 |
| Nov. | - | 22,000 | 0.4 |  |  | - | 17,600 | 0.4 |  | - | 27,500 | 1.10 | 38,500 | 2.20 |
| Dec. | - | 22,000 | 0.4 |  |  | $\checkmark$ | 22,000 | 0.4 |  | - | 27,500 | 1.00 | 38,500 | 2.00 |
| Jan. | - | 22,000 | 0.4 |  |  | - | 22,000 | 0.4 |  | - | 27,500 | 1.00 | 38,500 | 2.00 |
| Febr. | 900 | 22,000 | 0.4 | - |  | 900 | 22,000 | 0.4 |  | 900 | 27,500 | $\therefore 1.00$ | 38,500 | 2.00 |
| March | 3,850 | 22,000 | 0.4 |  |  | 3,850 | 22,000 | 0.4 |  | 3,850 | 27,500 | 1.10 | 38,500 | 2.20 |
| Apr. | 9,050 | 22,000 | 0.4 |  |  | 9,050 | 17,600 | 0.4 |  | 9,050 | 27,500 | 1.10 | 38,500 | 2.20 |
| Bay | 7,600 | 22,000 | 0.4 |  |  | 7,600 | 17,600 | 0.4 |  | 7,600 | 27,500 | 1.20 | 38,500 | 2.40 . |
| June. | 4,100 | 22,000 | 0.4 |  |  | 4,100 | 17,600 | 0.4 |  | 4,100 | 27,500 | 1.25 | 38,500 | 2.50 |
| July | 1,550 | 22,000 | 0.4 |  |  | 1,550 | 17;600 | 0.4 |  | 1,550 | 27,500 | 1.30 | 38,500 | 2.60 |
| Aug. | , - | 22,000 | 0.4 |  |  | - | 17,600 | 0.4 |  | - - | 27,500 | 1.35 | 38,500 | 2.70 |
| total | 117,000 |  |  |  | 93,600 |  |  |  | 146,250 |  |  |  | 204,850 |  |
| MAX | (1002) |  |  |  | (-20\%) |  |  |  | (+25\%) |  |  |  | (+75\%) |  |

### 4.2 Results

The results of the first run on the model indicated a much higher land utilisation than the prevailing one. The fallow rate at that time was nearly two thirds of the farmers' irrigable land, but the computer indicated that only $13.15 \%$ of the land should remain in fallow. The introduction of new fodder crops and of intensive milk production was responsible for this change. The obtained optimal production plan, as outlined in Table 12 , necessitates much higher levels of inputs than those prevailing, as indicated explicitly by the levels of the input purchasing activities such as water buying, labour hiring, and credit borrowing.

TABLE 12. The optimal production plan

| Husbandry | Dairy . | 6 cows + attached youngsters |
| :---: | :---: | :---: |
| Fodder crops | Silage maize - July | 0.566 ha |
|  | Fodder maize - June | 0.379 ha |
|  | Fodder maize - July | 0.331 ha |
|  | Fodder maize - August | 0.288 ha |
|  | Fodder maize - September | 0.289 ha |
| - . | * Fodder maize - October | 0.19 ha |
|  | Fodder maize - November | 0.328 hs |
|  | Alfalfa hay | 0.841 ha |
|  | SUPTOTAL | 3.213 ha $=24.71 \%$ of total land |
| Cash erops | Alfalfa hay | 1.758 ha |
|  | Cotton | 0.454 ha |
|  | Wheat | 2.261 ha |
|  | Sorghum | 3.606 ha |
|  | SUBTOTAL | 8.079 ha $=62.14 \%$ of total land |
|  | Total cultivated area | 11.292 ha $=86.86 \%$ of total land |
|  | Fallow | 1.708 ha $=13.15 \%$ of total land |
|  | Total land resources | 13.000 ha $=100.00 \%$ of total land |

The dairy enterprise entered the final solution as the dominating production branch, with an assumed maximum limitation of 6 cows. The VMP (Shadow Price) of this particular constraint reached a relatively high value of 4,459 Rls per cow. This value indicated that even at a lower efficiency, dairying would still maintain its comparative advantage in the optimum production plan; it also indicated the range of the permissible lower efficiency, or assuming the output levels of the model, lower milk prices.

The total amount of water and the composition of the monthly supply are outlined in Table 13. Highest water applications occur in the months of May and July.

TABLE 13. The optimal water supply plan

| Month | River and qanat water | Purchased well water | Total | Per cent |
| :---: | :---: | :---: | :---: | :---: |
| Sept. | - | 8,104 | 8,104 | 7.56 |
| Oct. | - | 7,944 | 7.944 | 7.42 |
| Nov. | * | 492 | 492 | 0.46 |
| Dec, | - | - | - | - |
| Jan. | $\cdots$ | - | - | - |
| Febr. | - | - | - | - |
| March | 3,850 | 8,180 | 12,030 | 11.23 |
| Apr. | 9.050 | 1,245 | 10,295 | $9.6!$ |
| May | 7.600 | 11.908 | 19,508 | 18.21 |
| June | 4,100 | 9,138 | 13,238 | 12.36 |
| July | 1,500 | 22,000 | 23,500 | 21.93 |
| Aug. | - | 12,025 | 12,025 | 11.22 |
| Total | $26,100 \mathrm{~m}^{3}$ | $81,036 \mathrm{~m}^{3}$ | 107,136 $\mathrm{ma}^{3}$ | 100.00 |

The obtained feeding plan is based on fresh green fodder maize (fed from June till November and grown on 6 successively sown plots of approximately 0.3 ha each), on silage maize, alfalfa hay, and bought concentrates. The inclusion of silage maize and alfalfa hay ensures flexibility and represents a rather simple feeding plan. The maximum monthly feeding restrictions of silage are "used up" in 7 out of the 12 months. Comparatively small efficiency gains in silage production may extend the use of this fodder, as can be concluded from the respective shadow pricés of the remaining 5 months.

The production plan of the field crops is dominated by the alfalfa hay selling activity, which is extended up to its full rotation constraiat. This crop was being grown extensively by Varamin farmers, who supplied the local demand of dairy farms in Varamin and also large dairy farms in the vicinity of Teheran. The future extension of this cash crop will be determined by the development of dairying around Teheran.

The shadow prices of the water supply restrictions used indicate the value of marginal product of this vital production resource. The highest value of 0.946 Rls per $\mathrm{m}^{3}$ indicates the upper limit of water costs and water prices. The following monthly values were obtained (Table 14).

TABLE 14. Monthly shadow prices of irrigation water

| Month | Shadow price (R1s/mi) |
| :--- | :---: |
| September | 0.508 |
| October | 0.459 |
| Hovember | 0.400 |
| March | 0.508 |
| Aprí | 0.459 |
| May | 0.400 |
| June | 0.400 |
| July | 0.946 |
| August | 0.643 |

The shadow price of the farmer family labour restrictions varied between 35 and 56 Ris per working day; thus the wages of $35-50 \mathrm{Rls}$ paid at present indicate. approximately the marginal productivity of this production factor. Table 15 summarizes the total net income obtained from the model farm.

TABLE 15. Summary of total net income

| Dairy . $\quad$. | 6 cows |  | - | $+$ | 94,450 R1s |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Silage maize - July | 0.566 ha | $\times$ | - 6,120.R1s/ha | - | 3,464 Rls |
| Fodder maize ~ June | 0.379 ha | $\times$ | - 5,110 R1s/ha | - | $1,936 \mathrm{Rls}$ |
| Fodder maize - July | 0.331 ha | $\times$ | - 5,110 R1s/ha | - | 1,691 R1s |
| Fodder maize - August | 0.288 ha | $x$ | - S,l10 R1s/ha | - | 1,471 Rls |
| Fodder maize - September | 0.289 ha | $\times$ | - 5,110 Rls/ha | - | 1,476 R1s |
| Fodder maize - Octaber | 0.191 ha | $x$. | - 5,110 Rls/ha | - | 976 Rls |
| Fodder maize ~ November | 0.328 ha | $\times$ | - 5,110 Rls/ha | - | 1,676 R1s |
| Alfalfa hay fodder | 0.841 he | $\times$ | - 15,500 R1s/ha | - | 13,035 R1s |
| Alfalfa hay seiling | 1.758 ha | $\times$ | 22,300 Rls/ha | + | 39,203 R1s |
| Cotton | 0.454 ha | $\times$ | 27,900 Rls/ha | + | 12,666 R15 |
| Wheat | - 2.261 ha | $\times$ | 16,220 Rls/ha | + | 36,673 Rls |
| Sorghum | 3.606 ha | . $\times$ | 15,520 Rls/he | + | 55,965 R1s |
| Capital borrowing | 64,482 R1s | $\times$ | - 0.03 RIs | - | 1,934 R1s |
| Labour hiring | 659.2 L.D | $\times$ | - $35.00 \mathrm{Rls} / \mathrm{L} . \mathrm{D}$ | - | 23,072 R1s |
| Water buying | $81,036 \mathrm{~m}^{3}$ | $\times$ | - $0.4 \mathrm{RLs} / \mathrm{m}^{3}$ | - | 32,414 R1s |
| Total, net income |  |  |  | + | 155,812 R1s |

In the parametric runs showing the impact of increasing interest rates on capital, the following solutions were obtained (Table 16):

TABLE 16. Impact of higher interest rates

| Activity | 6\% | 10\% | 15\% | - $20 \%$ |
| :---: | :---: | :---: | :---: | :---: |
| Daitry | 6 cous | 6 cows | 6 cows | 6 cows |
| Silage maize - Juiy | 0.566 ha | 0.566 ha | 0.566 ha | 0.566 ha |
| Fodder maize - June | 0.379 fa | 0.379 ha | 0.379 ha | 0.379 ha |
| Fodder maize - July | 0.331 ha | 0.331 ha | 0.331 ha | 0.331 ha |
| Fodder mizize - Aug. | 0.288 ha | 0.288 ha | 0.288 ha | 0.288 ha |
| Fodder maize - Sept. | 0.289 ha | 0.289 ha | 0.289 ha | 0.289 ha |
| Fodder maize. - Oct. | 0.191 ha | 0.191 ha | 0.191 ha | 0.191 ha |
| Fodder maize - Nov. | 0.328 ha | 0.328 ha | 0.328 ha | 0.328 ha |
| Alfalfa hay fodder | 0.841 ha | 0.84 ] ha | 0.841 ha | 0.841 ha . |
| Alfalfa hay selling | 1.758 ha | 1.758 ha | 1.758 ha | 0.483 ha |
| Cotton | 0.454 ha | 0.684 ha | 0.684 ha | 1.784 ha |
| Wheat | 2.261 ha | 1.693 ha | 1.693 ha | 0.872 ha |
| Sorghum | 3.606 ha | 2.531 ha | 2.531 ha | 1.194 ha |
| Maize grain | - | 0.768 ha | 0.768 ha | 1.863 ha |
| Capital borrowing | 64,482 Rls | 58,336 Rls | $58,336 \mathrm{kls}$ | 49,875 Rls |
| Labour hiring | $659.2 \mathrm{L.D}$ | $670.7 \mathrm{L.D}$ | $670.7 \mathrm{L.D}$ | $693.2 \mathrm{L.D}$ |
| Water buying | $81,036 \mathrm{~m}^{3}$ | 82;039 m ${ }^{3}$ | 82,039 m ${ }^{\text {3 }}$ | $79,840 \mathrm{~m}^{3}$ |
| Total Net Income | 155,812 Rls | 145,906 Rls | 141,069 R1s | 127,115 R1s |
| Percentage | 100.00 | 93.64 | 90.54 | 81.58 |
| Change in net income ${ }^{\text {l }}$ |  | -9,906 | - 14,743 | - 28,697 |
| Change of interest | costs ${ }^{1}$ | + 983 | $+2.441$ | + 3,053 |

1 Compared with pesults of the master matrix: $6 \%$ interest rate

On1y the rate of interest was changed in these runs; all the other data meet the required condition of "ceteris paribus". A rise in interest rates can have two effects:
a) substitution of production factors
b) declining income.

The first effect is generally followed by a change in the production plan. The increase from $6 \%$ to $10 \%$ caused some minor changes in the cash crop production plan. Wheat was very vulnerable, sorghum was partly replaced by maize grain. The income loss was $9,906 \mathrm{Rl}$ s and the additional interest costs were 983 Rls .

The next step from $10 \%$ to $15 \%$ demonstrates the pure case of an income effect as no changes in production plans and resource use took place.

The last step from $15 \%$, to $20 \%$ caused a decrease in crops like wheat and alfalfa, which are capital-intensive crops with a comparatively long production season. Labour continued to substitute for capital. Dairying and its fodder supply remained unaffected.

This analysis indicates the impact and potential of an adequate credit supply policy by the government and banking institutions, as it clearly differentiates productivity sacrifices and income declines caused by raising credit costs.

The purpose of these parametric runs was to assess the quantitative impact of various water supply policies. The policy of Alternative 2 was defined as a $20 \%$ restriction of well water extraction during the period April to November and as a total decrease in water allotments of $20 \%$. Well water prices remained unchanged at $0,4 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$. The impact of this policy was quite moderate, only small decreases in cash crops taking place (Table 1.7).

The dairy branch was not affected. The net income loss amounted to $24,631 \mathrm{Rls}$ (16\%) . This alternative might emerge now that part of the Jaj Rud Water is being diverted to Teheran. It must be causing substantial losses to the average varamin farmer, because most of them do not maintain an efficient dairy branch as included in the model.

Alternative 3, more but costlier water, caused a rapid decline in production and only the dairy branch remained at its full level of 6 cows. The cash crops all but disappeared from the production programae. In this alternative the marginal water price of $1.0 \mathrm{Rls} / \mathrm{m}^{3}$ exceeded the marginal value of a $\mathrm{m}^{3}$ water as calculated by the model to be $0.946 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$. The latter value was reached in one month only, namely in July, while during the other months its upper limit did not exceed $0.645 \mathrm{Rls} / \mathrm{m}^{3}$. Thus, the higher water prices would cause a substantial decrease in agricultural production and consequently also in the farmer's income.

TABLE 17. Solutions of the four alternatives of water supply


The fourth alternative caused almost no production of any kind, as only one crop, sugar melon, could pay for the high water price of $2.0 \mathrm{Rls} / \mathrm{m}^{3}$. It can be concluded, therefore, that the maximum water price, or the maximum water supply cost, disregarding for the moment any subsidizing policies, should not exceed $1.0 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$.

The sensitivity tests of the assumed water requirement coefficients were based on a range of $20 \%$ higher or lower requirements (Table 18 ). The results obtained could also be interpreted as the possible impact of future irrigation efficiencies on the optimal agricultural production programme. These results reinforce the previous conclusion as to the stability of including the dairy branch and its associated fodder crops in the progranme. These branches were not affected by the higher or lower water consumption. Likewise, the comparatively profitable alfalfa hay selling activity maintained its optimal size of 1.758 ha. Cotton showed a rather high sensitivity in its water requirements: efficiency gains

TABLE 18. Results of sensitivity tests of various water consumption coefficients

| Water coefficients | -20\% | , | Mester matrix | +20\% |
| :---: | :---: | :---: | :---: | :---: |
| Dairy | 6 cows |  | 6 cows | 6 cows |
| Siłage maize - July | 0.566 ha |  | $0.56{ }^{\circ} \mathrm{ha}$ | 0.566 ha |
| Fodder maize - June | 0.379 ha |  | 0.379 ha | 0.379 ha |
| Fodder maize - July | 0.331 ha |  | 0.331 ha | 0.331 ha |
| Fodder maize - August | 0.288 ha |  | 0.288 ha | 0.288 ha |
| Fodder maize - September | 0.289 ha |  | 0.289 ha | 0.289 ha |
| Fodder maize - October | 0.191 ha |  | 0.191 ha | 0.191 ha |
| Fodder maize - Novenber | 0.328 ha |  | 0.328 ha | 0.328 ha |
| Alfalfa fodder hay | 0.841 ha |  | 0.841 ha | 0.841 ha |
| Alfalfa hay selling | 1.758 ha |  | 1.758 ha | 1.758 ha |
| Cotton | 1.380 ha |  | 0.454 ba | 0.257 ba |
| Wheat | 1.944 ha |  | 2.261 ha | 1.378 tha |
| Sorghum | 3.203 ha |  | 3.606 ha | 2.070 ha |
| Maize grain | 1.126 ha |  | - | 0.493 ha |
| Sugar melon | 0.116 ha |  | - | - |
| Capital borrowing | 76,416 Rls |  | 64,482 Rls | 46,205 Rls |
| labour hiring | 917.7 L.D. |  | $659.2 \mathrm{~L} . \mathrm{D} .$ | $511.6 \text { L.D. }$ |
| Water buying | 78,132 m ${ }^{3}$ |  | $\mathrm{B} 1,036 \mathrm{~m}^{3}$ | 86,982 m ${ }^{3}$ |
| Net income | 184,924 R1s |  | 155,812 R1s | 124,424 R1s |
| Total water quantity | 108,942 m ${ }^{3}$ |  | 107,136 m ${ }^{3}$ | 116,030 m ${ }^{3}$ |
| Max. VMP/m ${ }^{3}$ | 1.260 |  | 0.946 | 0.579 |

of $20 \%$ would increase the cotton area to $304 \%$, compared with the average in the matrix, while the $20 \%$ higher consumption figure would cause a contraction of the cotton area to $57 \%$. The changes obtained in hectarage of the remaining cash crops clearly indicate the differing substitution races of these crops, from the point of view of their water consumption. Thus,with the lower consumption, wheat would partly be substituted by maize grain and sugar meion, while with the higher consumption, the wheat and sorghum area would decrease and be partly replaced by grain maize.

A higher irrigation efficiency would increase the value of marginal product of $\mathrm{a}^{3}$ water from $0.946 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$ to $1.260 \mathrm{Rls} / \mathrm{m}^{3}$, an increase of $33.2 \%$. This conclusion could have important consequences for water development.costs and water subsidies as interim measurcs during the first years of project implementation, i.e. until the farmer has obtained a higher irrigation efficiency and is thus able to bear the full water costs.

### 4.3 Conclusions

The optimized agricultural production plan obtained from the linear programming model clearly emphasizes the development of the dairy branch, and thus defines the optimal farm type for the Varamin Ylain as a "Dairy + Cash Crop" farm. However, the inclusion of dairying in the future development programme of the Varamin Plain must be carefully planned and implemented, in view of the high levels it requires of investment and managerial abilities. Development could take place in two phases. In the first, efforts could be concentrated on the already existing dairy farms: the milk collection system could be improved, artificial insemination could be more widely applied, the proposed new fodder crops could be introduced, as well as modern ensilaging techniques. Simultaneously a new government policy on long-term planning for the various milk production areas intended to supply the growing demand of Greater Teheran should be crystallized and provide clear guidelines for the further development of the dairy branch in the Varamin project. In the second phase, new additional dairy farms could be developed, and the markering and processing of milk and dairy products from the Plain could be further modernized.

These considerations will lead to the area of irrigated fodder land being less than the optimized one. The dominating position of the dairy branch merely indicates an economically sound development target, which could only be realized, step-by-step, over a longer development period.

The development of the proposed irrigated summer grain crops, such as sorghum, maize, but also sunflower and safflower, had already been tested in various parts of the Plain. Their future expansion will also be a gradual one, depending largely on the experience gained and on the ability of extension officers to convince reluctant and tradition-bound farmers to try these new crops. The completion of the new water supply and irrigation systems will undoubtedly ease their efforts, as irrigation of these crops in summer will then be possible.

Production and sales of alfalfa hay will also remain one of the most profitable activities in the forseeable future. Further investments in commercial dairying around Teheran will increase the demand for this product, as will the recent establishment of new and modern catcle and sheep fattening farms around the capital. Some uncertainty, however, surrounds the future expansion of this trade, as more and more dairymen are tending to enter into long-term supply contracts with modern, large-scale, and mechanized alfalfa hay producers, or to rent land and ptoduce their own hay. It would be advisable to make a careful analysis of further developments in the alfalfa hay market and also to investigate the feasibility of cooperative marketing and processing.

The development of the vegetable production branch in the Varamin Plain must be promoted with extreme care. In spite of its geographical proximity to the principal market of the country, the Teheran wholesale market, the Varamin vegetable producers must compete with producers from other regions, even including the comparatively far-off Khuzistan region, which supplies fresh vegetables in specific seasons from a distance of more than $1,000 \mathrm{~km}$. Various new irrigation projects in this and other areas of the country, such as Ghazvin or Shiraz, and the rapid improvement of the country's transportation and marketing systems, will endanger the present comparatively advantageous position of the Varamin vegetable producer.

The subject of regional specialization of agricultural production in Iran was being extensively studied to find adequate guidelines for optimized regional development programmes. Future industrialization-and urbanization, which can already be foreseen within the proposed establishment of new basic industries in various parts of the country, will undoubtedly alter the traditional production pattern and marketing channels. These studies may also reveal specific insights into the comparative advantage of the Varamin Plain. It is thus conceivable that in the future the Plain might concentrate on the supply of high-value perishables to the capital and thus gradually phase out as a cotton and wheat producer.

Such possibilities, combined with those of the dairy industry, call for a rather flexible agricultural production programe, one which could easily be amended without sacrifices of previous long-term investments.

The obtained optimal agricultural production plan must of course be transposed into the reality of its practical implementation on the farm. Thus, the precisely calculated production areas of, e.g. 0.289 ha fodder maize, muse be rounded up to manageable sizes.

The following rotation programe (Table 19) has been elaborated in order to demonstrate this phase of the planning procedure.

TABLE 19. Land use and rotation plan

| Total available land | 13.000 ha |
| :--- | ---: |
| Perennial alfalfa crops: |  |
| a) for own hay feeding <br> b) for hay selling | 0.841 ha |
| TOTAL | 1.759 ha |
| Remaining land for rotation | 2.500 ha |

Rozation plan

| Ist year | cotton | 0.454 ha |
| :--- | :--- | :--- |
|  | silage maize | 0.566 ha |
|  | fodder maize | 1.806 ha |
|  | fallow | 0.674 ha |
|  | TOTAL | 3.500 ha |
| 2nd year | sorghum | 3.606 ha (say, 3.500 ha) |
|  | wheat | 2.261 ha |
|  | fallow | 1.239 ha |
|  | TOTAL | 3.500 ha |

### 4.4 Subregional agricultural production patterns

The prototype of optimal land use patterns described above was incorporated into the final planning considerations of the project. These considerations were also based on the relatively large amount of new data accumulated by the project team during their stay. These data included the results of a detailed socioeconomic survey, an updated soil survey, and data from agricultural experimentation and demonstration plots in various areas of the Varamin Plain.

In view of this additional information it became necessary to redefine and synthesize optimal agricultural production patterns, taking into consideration the subregional specialization. It also became evident that some existing land use patterns, mainly orchards of figs, pomegranates, and grapes, as well as vegetable growing, should be continued, as they represent not only an excellent utilization of the land and water resources in their particular subregions, but also a long tradition of specialization in various villages, which could not and certainly also should not be discontinued abruptly.

In line with these considerations five homogeneous production areas were defined (Fig.27), homogeneous from the point of view of soil properties, existing crop patterns,microclimate,etc. An optimal land use pattern was calculated for each of these zones, representing a synthesis of the results of the linear programming and the additional factors outlined above.

Table 20 shows the cropping patterns for the five zones, and compares them with the linear programming solution. The dairy enterprise has been included, but on a reduced level, while cotton, the main traditional cash crop, will maintain its present dominant position. The new summer grains are mainly planned for zones $B$ and $E$, as the others will continue their relative specialization in orchards or vegetables.

Based on the additional data, new coefficients were calculated and used in updating the various crop budgets. Table 21 shows these crop budgets, in terms of per ha values of output, input, net income, water requirement coefficients and net income per $\mathrm{m}^{3}$ of water. These values were used for determining the polygonal demand for irrigation water, and represent in each zone and each polygon the average net income per $\mathrm{m}^{3}$, based on the specific production pattern of each zone (Table 22 and Fig.27).


TABLE 20. Proposed crop patterns for the five zones compared with the linear programang solution (in 8 of the cropping area)

| Zone | A | B | C | D | E | LP MODEL |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| CROP |  | - |  |  |  |  |
| Cotton | - | 43.6 | 43.7 | 33.1 | 43.4 | 4.02 |
| Wheat | - | 20.3 | 19.6 | 10.4 | 20.0 | 20.02 |
| Alfalfa | - | 7.2 | 10.5 | - | 7.2 | 23.01 |
| Haize grain | - | 7.7 | - | - | 10.4 | - |
| Maize fodder | - | 7.1 | - | - | 9.5 | 21.02 |
| Safflower | - | 14.3 | - | - | 9.5 | - |
| Cantalloup | - | - | 26.2 | - | - | $31.93^{1}$ |
| Tomato | - | - | - | 29.4 | - | - |
| Cucumber (spring) | - | - | - | 10.2 | - | - |
| Cucumber (fall) | - | - | - | 4.2 | - | - |
| Green vegetables ${ }^{2}$ | 40.8 | - | - | 7.3 | - | - |
| Orchards | 59.2 | - | - | 5.4 | - | - |
| TOTAL | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

[^1]TABLE 21. Net income per $\mathrm{m}^{3}$ of water for various crops

| Crop | Value of output | Yaiue of input for |  |  | Tocal | Net incotere | Water requirement $\left(m^{3} / h a\right)$ | Net income$\left(\mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | maceríal $\qquad$ (R | 1abour <br> ials / h | machinery |  |  |  |  |
| Cotton | 50,000 | 5,936 | 12,800 | 3.154 | 21,890 | 18,110 | 13,000 | 2.162 |
| Safilower | 22,500 | 4,585 | 4,200 | 6,007 | 14,792 | 7,708 | 7,000 | 1.101 |
| Alfalfa | 144,000 | 23,256 | 48,800 | 5,100 | 77.156 | 66,844 | 16,000 | 4.178 |
| Sorghum | 27.750 | 8.056 | 6.900 | 3,037 | 17.993 | 9,757 | 9,000 | 1.084 |
| Cantaloup | 60,500 | 13,356 | 8,800 | 3,047 | 25,203 | 35,297 | 9,400 | 3.755 |
| Tomatocs | 120,000 | 28,355 | 24, 100 | 3,302 | 55,757 | 64,243 | 11,600 | 5.538 |
| Haize | 31.250 | 9,952 | 6,900 | 3,037 | 19,889 | 11,361 | 10.800 | 1.052 |
| Wheat | 20,800 | -3,471 | 4,400 | 2,143 | 10,014 | 10,786 | 7,000 | 1.54 I |
| Sunf lower | 27,000 | 4,770 | 5,600 | 3,037 | 13,407 | 13,593 | 10,000 | 1.359 |
| Cucumber (spring) | 75,000 | 13,728 | -14,700 | 2,958 | 31,386 | 43,614 | 10,000 | 4.361 |
| Cucumber (fall) | 27.000 | 6.572 | 5.600 | 1,484 | 13,656 | 13,344 | 6,000 | 2.224 |

TABLE 22. Average net income per $\mathrm{m}^{3}$ of each agricultural production zone

| Zone | Polygon | Crop pattern | $\%$ | ha | $\mathrm{Rls} / \mathrm{m}^{3}$ | $\mathrm{m}^{3} / \mathrm{ha}$ | Average |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | $\mathrm{R1g} / \mathrm{m}^{3}$ | $m^{3} / \mathrm{ha}$ |
| A | 1 | orchards | 59.2 | 200.1 | 2.000 | 13,400 | 1.184 | 7,932 |
|  |  | vegetables | 40.8 | 137.9 | 2.500 | 10,000 | 1.020 | 4,080 |
|  |  | TOTAL | 100.0 | 338.0 | - | - | 2.204 | 12,012 |
| B | 2,3,7,8,9 | cotton | 43.6 | 6,641 | 2.162 | 13,000 | 0.943 | 5,668 |
|  |  | wheat | 20.1 | 3,062 | 1.541 | 7,000 | 0.310 | 1,407 |
|  |  | alfalfa | 7.2 | 1,097 | 4.187 | 16:000 | 0.301 | 1,152 |
|  |  | maize | 7.7 | 1,173 | 1.052 | 10,800 | 0.081 | 832 |
|  |  | fodder maize | 7.1 | 1,083 | 3.052 | 10,800 | 0.075 | 767 |
|  |  | safflower | 14.3 | 2,178 | 1.101 | 7,000 | 0.157 | 1,001 |
|  |  | TOTAL | 100.0 | 15,232 | - | - | 1.867 | 10,827 |
| C | 4,5,6,10,11 | Cotton | 43.7 | 4,267 | 2.162 | 13,000 | 0.945 | 5,681 |
|  |  | wheat | 19.6 | 1,964 | 1.541 | 7,000 | 0.302 | 1,372 |
|  |  | cantaloup | 26.2 | 2,558 | 3.755 | 9,400 | 0.984 | 2,463 |
|  |  | alfalfa | 10.5 | 1,025 | 4.178 | 16,000 | 0.439 | 1,680 |
|  |  | total | 100.0 | 9,764 | - | - | 2.670 | 11,196 |
| D | 12,13,14 | cotton | 33.1 | 1.877 | 2.162 | 13,000 | 0.716 | 4,303 |
|  |  | wheat | 10.4 | 590 | 1.541 | 7,000 | 0.160 | 728 |
|  |  | tomato | 29.4 | 1,667 | 5.538 | 11,600 | 1.628 | 3,410 |
|  |  | cucumber-spring | 10.2 | 578 | 4.361 | 10,000 | 0.445 | 1,020 |
|  |  | cucumber-fall | 4.2 | 238 | 2.224 | 6,000 | 0.093 | 252 |
|  |  | vegetables | 7.3 | 414 | 2.500 | 10,000 | 0.183 | -730 |
|  |  | orchards | 5.4 | 306 | 2.000 | 13,400 | 0.108 | 724 |
|  |  | TOTAL | 100.0 | 5,670 | - | - | 3.333 | 1],167 |
| E | 15 to 23,26 | cotton | 43.4 | 11,493 | 2,162 | 13,000 | 0.938 | 5,642 |
|  |  | whest | 20.0 | 5,296 | 1.541 | 7,000 | 0.308 | 1,400 |
|  |  | maize | 10:4 | 2,754 | 1.052 | 10,800 | 0.109 | 1,123 |
|  |  | fodder maize | 9.5 | 2,516 | 1.052 | 10,800 | 0.100 | 1,026 |
|  |  | safflower | 9.5 | 2,515 | 1.101 | 7,000 | 0.105 | 665 |
|  |  | alfalfa | 7.2 | 1,907 | 4.178 | 10,000 | 0.301 | 1,152 |
|  |  | TOTAL | 100.0 | 26,481 | - | - | 1.861 | 11,008 |

For each of the five zones, the average net income per $m^{3}$ and the average annual water requirement per ha are summarized in Table 23.
table 23. Summary of the average net income per $\mathrm{m}^{3}$ and the average annual water requirement per ha of the five distinguished zones of agricultural production

| Zone | $\begin{gathered} \text { Average net } \\ \text { income } \\ \mathrm{Rls} / \mathrm{m}^{3} \end{gathered}$ | ```Average water requirement m}/\textrm{ha``` |
| :---: | :---: | :---: |
| A | 2.204 | 12,012 |
| B | 1.867 | 10,827 |
| C | 2.670 | 11,196 |
| D | 3.333 | 11,167 |
| E | 1.861 | 11,008 |

## 5. Developing a linear programming test model

Owing to the complexity of the problems we were facing in using a linear programming model to aid us in planning an optimal irrigation water supply system, we decided that we should first develop a simple test model (EREZ, 1967). In a later stage of the study, we found that some of the model's components and some of the assumptions made were erroneous and had to be redefined. Even so, the test model proved to be of great help in developing the ultimate comprehensive linear programming model.

One of the reasons why we decided to apply linear programming in an early stage of the project is the major shortcoming of the traditional approach, which is characterized by an independent and usually insufficiently coordinated search by each discipline for a technically feasible solution to the problem. Thus, the hydrologist, irrigation engineer, agronomist, and other members of the project team are elaborating and generating their specific data and proposals, assuming that their proposals represent an optimal solution which has only to be incorporated into the final and comprehensive proposals of the team.

This approach can result in sub-optimal solutions, which may be feasible technically,but are economically unjustified,because the various technical disciplines may opt for capital-intensive, elegant, and technologically advanced solutions, believing somewhat a priori in their economic superiority. Afterwards it becomes the economist's plight to prepare the cash flow analysis and to produce the economic justification for the proposed solution(s). In awareness of this problem, an early attempt was made to develop and use a linear programming model that would include all the various activities, assuming. one single target function: an optimized net income of the farmers of the Varamin Plain, which are thought to be represented by one single water supply authority working on a non-profit basis.

### 5.1 The test model

A prerequisite for developing a linear programming water supply model whose results, generated by a computer, can be directly tested by a groundwater simulation model, is that the linear progranming model be grafted on the groundwater
model. For this purpose the network of polygons used must be the same as that designed for the groundwater model. The agricultural activities of the farmers were defined in the test model as polygonal production activities, iPROD, where $i=p o l y g o n .1,2, \ldots n$. The costs of these activities represented the net income per $\mathrm{m}^{3}$ of water, interpreted also as the maximum price of water that farmers could afford to pay in Polygon i. Interpolygonal water price equalizations or general water price subsidies were not included in the model.

The various activities to supply water to a given polygon $i$ were defined in the test model in accordance with the existing supply practices in the Plain and with the planning opinions of the project team members as these were put forward early in the project.

These activicies were:

| a) | Groundwater pumpage from existing wells in Polygon i | itELI |
| :---: | :---: | :---: |
| b) | Groundwater pumpage frotu new and deeper wells in Polygon i | iWEL, 2 |
| c) | Groundwater supply to Polygon $j$ by a qanar withdrawing from Polygon i | $\mathrm{i} G \mathrm{Nij}$ |
| d) | Substitution of a qanat in Polygon $i$ (drying out of a qanat) by pumping mare fron existing andor new wells | iSUBG |
| e) | Supply of surface water to Polygon i | isurf |
| f) | Supply of imported surface water to Polygon i | IIMP0 |
| g) | Transportation of well water by canals from Polygon ito Polygon $\mathbf{j}$ | iWTRj |
| b) | Artificial recharge in recharge basin $A$ | RECHA |
| i) | Artificial recharge in recharge basin $B$ | RECHB |
|  | Artificial recharge in recharge basin $C$ | RECHC |

The principal aim in this phase of the work was merely to develop a test model from which experience could be gained as to how the ultimate model could best be organized and developed. For reasons of simplicity and to save time and computer costs, the rest model was not yet grafted on the groundwater model.

The Varamin Plain was divided into five imaginary polygonal areas, instead of the 27 used for the groundwater model (Fig. 28). Dummy data for the costs and returns of the various activities were used, representing the best estimates of the real magnitude of supply costs and net benefits that team members could provide at this stage of the project. The three activities of artificial recharge of the groundwater were differentiated for three basins of different location, size, and costs..-


Fig. 28. Linear Prograrming Water Supply Test Model.

The constraints used in this simplified test model were:

| a) "Safe yield" of the groundwater resource in Polygon i | iSFYL |
| :--- | :--- | :--- |
| b) Maximum capacity of groundwarer pumpage from existing wells | irXWL |
| in Polygon $i$ |  |

The coefficients of che matrix represented water percolation, water losses by evaporation, and interpolygonal groundwater flow, all resulting in water table changes.

In the actual situation, groundwater pumpage from a certain polygon will cause the water table to drop in that polygon, but also in adjacent polygons, though less than in the polygon in question. In other words the total groundwater pumpage from a certain polygon is made up of a certain quantity withdrawn from that polygon and certain quantities withdrawn from adjacent polygons. In the test model we had to account for this phenomenon. We did so by assuming certain groundwater extraction ratios. These ratios are presented in Table 24 , but it should be realized that they are of a purely hypothetical nature.

TABLE 24. Hypothetical fractional contribution of the various polygons for unit volume of groundwater pumpage in a particular polygon


The above fractions state that the pumpage of one million $\mathrm{m}^{3}$ groundwater from polygon 1 will draw from the "safe yield" of polygon 1 ( 0.4 million $\mathrm{m}^{3}$ ), polygon 2 ( 0.4 million $\mathrm{m}^{3}$ ) and polygon 3 ( 0.2 million $\mathrm{m}^{3}$ ).

Another simplification of the test model enforced a certain sequence of groundwater pumpage activities:

1. pumpage from existing wells and/or existing qanats
2. drying up of the qanats by additional pumpage from existing wells
3. pumpage from new wells.

The effect of these groundwater extraction activities on the water table and the extraction costs were thus incorporated in the model. Table 25 shows the linear programming matrix of the model.

### 5.2 Results obtained from the test mode!

As stated earlier, this simplified, or rather oversimplified, test model provided guidelines along which the development of the major linear programming model had to be organized. One of the basic problems of the Varamin Plain in need of solution was that of allocation. Should farmers, wherever they were Iiving, be supplied with irrigation water by the new and modern supply system, or should the scanty and costly water be supplied only to the fertile soils of the Plain? The settlement of farmers on these soils, which are now only partly used but which can be brought under irrigation, therefore had to be considered. This resettlement process could be regarded either as a continuation of the


Tab.25. LP matrix of test model.
historical migration process in the Plain, where farmers over the last 30 years have gradually been abandoning the peripheral villages and hamlets, or as a properly planned and financed project activity that should be incorporated in the new development programe for the Plain.

The linear programming test model provided a very explicit answer to the water allocation problem, as can be seen from Table 26 , which shows the optimal allocation of the limited water resources.

TABLE 26. Linear programming solution of water supply

| Activity | Polygon number | Net return per ${ }^{3}$ (Rls) | Maximum capacity (iMKDM) | Optimal use of the capacity |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | IPROD | Percentage |
| 1 PROD | 1 | 3.71 | 159.3 | 159.3 | 100 |
| 2 PROD | 2 | 1.71 | 118.0 | 118.0 | 100 |
| 3 PROD | 3 | 1.30 | 100.7 | 91.5 | 90.9 |
| 4 PROD | 4 | 0.90. | 93.5 | 25.4 | 27.2 |
| 5 PROD | 5 | 0.50 | 19.4 | - | - |

The solution suggests that polygon 5 should be abandoned and 100 per cent of the Class I and Class LI soils in polygons 1 and 2 should be utilized.

This solution is rather versatile, but complex in that it is made up of an integrated system of qanats, existing wells and new wells, the drying up of one qanat, the concentrated supply of surface water to polygons 1 and 3, and the importation of surface water in polygon 1 , as can be seen from Table 27.

TABLE 27. Linear programming solution of water supply differentiated for the various water resources

| Polygon <br> number | irXXH | iPROD | Surface <br> water | Well <br> water | Qanat <br> water | Total <br> (million $\mathrm{m}^{3}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 159.3 | 159.3 | 131.2 | 28.1 | - | 159.3 |
| 2 | 118.0 | 118.0 | - | 114.0 | 4.0 | 118.0 |
| 3 | 100.7 | 91.5 | 31.7 | 34.8 | 25.0 | 91.5 |
| 4 | 93.5 | 25.4 | - | - | 25.4 | 25.4 |
| 5 | 19.4 | - | - | - | - | - |
| Total | 490.9 | 394.2 | 162.9 | 176.9 | 54.4 | 394.2 |

The solution revealed the rather low returns per $m^{3}$ in polygons 4 and 5 , where polygon 4 could "afford" only the low cost qanat water, all other water resources being either too costly or having higher opportunity costs in other polygons. It also became obvious that the full irrigation of all the potentially irrigable land would be uneconomic, given the assumed cost-return ratio.

The artificial recharge activities do not appear in the solution owing to their comparatively high bosts and their alternative use possibilities, as defined in the test model. Their economic feasibility should be analyzed separately, based on the accumulated experience gained with recharge basins where costs, infiltration rates, evaporation losses, optimal design, etc., can be studieJ.

The initial idea of including the qanats in a modern and fully integrated water supply system had to be rejected because of their uncontrollable flow and resulting wastage. These ancient groundwater extraction structures will be replaced by modern tube wells. The additional new wells will cause the water table to drop, resulting in the drying up of most of the still discharging qanats.

The test model was developed on the basis of annual quantities of water availability and water demand; in other words, the divergencies between the monthly demands for irrigation water and the monchly availability of surface water were disregarded. However, for an optimized agricultural production programme for the Plain, such a monthly water demand function was estimated in two ways:
a) a linear programming model (see Chap. 6 and EREZ, 1967)
b) normative data of three proposed farm types for the Plain (see Table 28 and EREZ, 1967)

The two demand functions indicated the same surface water surplus in early spring (March - June) and a substantial deficit in the sumer months (see also Table 6). The optimal solution to this discrepancy problem will have to be derived in the future from the following three complementary activities:
a) regulating the river flow by constructing a second dam
b) utilizing the groundwater basin's storage capacity and surplus river flows to recharge the basin artificially
c) introducing flexible cropping patterns and irrigation techniques.

TABLE 28. Monthly irrigation water demand and supply functions

| Month | Demand function of the linear programing model | Aggregated demand function of the 3 farm types | Discharge of the Jaj Rud ${ }^{1}$ | Demand in per cent of the river discharge |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1in.progr. model | 3 farm types |
| * | \% | . 7 | 7 | 7 | $\%$ |
| September | 7.56 | 9.40 | 2.90 | 261 | 324 |
| October | 7.41 | 10.00 | 2.85 | 260 | 351 |
| November | 0.46 | 3.20 | 3.65 | 13 | 88 |
| December | - | 3.30 | 3.56 | - | 93 |
| January | - | - | 3.28 | - | - |
| February | - | - | 4.09 | - | - |
| March | 11.23 | 8.00 | 7.86 | 143 | 102 |
| April | 9.61 | 10.50 | 18.74 | - 51 | 56 |
| May | 18.21 | 17.10 | 26.63 | 68 | 64 |
| June | 12.36 | 14.50 | 15.38 | 80 | 94 |
| July | 21.94 | 14.30 | 6.98 | 314 | 205 |
| Auguet | 11.22 | 9.70 | 4.08 | 275 | 238 |
| TOTAL | 100.00 | 100.00 | 100.00 | - | - |

1 Average of 20 beans

The possibilities of constructing a (second) regulatory dam downstream of the confluence of the Jaj Rud and Damavand Rud were investigated, but appear to be very limited due to the lack of suitable sites and sufficiently large storage capacities.

The utilization of the groundwater basin for storage and for additional extraction in surface water deficient years or periods have been discussed in Chapter 2, Section 8.

It is an ancient practice of the farmers in the Varamin Plain to adapt the area of their irrigated land to the expected Jaj Rud flow in the current agricultural year. Similarly they decide on the timing of land preparation and early spring irrigation. This practice is based on regular observations of the extent and thickness of the snow cover on the Elburz Mountains facing the Varamin Plain. The snow cover is an indication of the Jaj Rud flow that may be expected in spring and early summer. Hence, an extensive and chick snow cover, observed in January and February, may induce the farmers to prepare more land for irrigation than they would if only little snow is seen on the mountains.

It seems conceivable that such a flexible adaptation of the area of irrigated land to the expected river flow could also be applied in the future, to grow, for instance, summer grain, oil seed or fodder crops. It should, however, be based on more refined and scientific measurenents of the extent, thickness, and consistency of the snow cover in the catchment area of the Jaj Rud. These measurements could be made weekly by the proposed regional development authority. In this way farmers could be advised in proper time about the surface water expected to be available in the coming year, and can then adapt their cropping programmes accordingly. Notice of 6 to 8 weeks may be sufficient for this purpose.

As stated earlier, the results obtained from the test model allowed seyeral valuable conclusions to be drawn which were later used in developing the comprehensive linear programming water supply model in combination with the groundwater simulation model. It became clear, for instance, that methodologies had to be developed to test the impact on the regional water table of the various groundwater pumpage patterns calculated by the linear programing model and to specify the percolation coefficients and percolation losses from the proposed canal system. It also became evident that a specific agricultural production programe had to be developed for each polygonal area, based on its soil properties and also taking into consideration the prevailing cropping systems.

## 6. Developing the comprehensive linear programming model

The basic constraints of our problem were that the available water was limited and that the water needs of the agricultural production schedule had to be met, while our objective was to maximize the farmer's net income. What we had to find was a solution that satisfied the two constraints while achieving our objective: in other words, we had to find an optimum solution.

Since our primary concern was to supply irrigation water economically, our problem was one of economic optimization. The cost of water differs in different parts of the Plain. A canal system is needed to convey surface water from the diversion weir in the river, and the farther the water has to be conveyed, the higher the costs will be. Similarly, the cost of well water differs: in the south the water table is less than 10 m deep and groundwater recovery is reasonably cheap; towards the north it is 60 to 80 ml deep and, because the higher lift requires more energy, costs are higher.

It is beyond the scope of this publication to present an exposition on the techniques of linear programming that we applied to our problem. Many text books have been written on the subject, including those by GASS (1958), GARVIN (1960), AN-MIN-CHUNG (1962), and SPIVEY (1963), to which the reader is referred for more information.

Basic ro developing the comprehensive model was that it be grafted on the network of polygons constructed for the groundwater simulation model. This meant that all activities like agricultural production, surface water supply, groundwater supply, and their costs and net returns, as well as the constraints like water requirements, maximum water demands, and available water, had to be expressed on a polygonal basis, i.e. on the basis of the sub-areas into which the Plain had been divided for the groundwater model (Fig.19).

Linear programing generally has three quantitative components: an objective, alternative activities for achieving that objective, and resource or other restrictions.

### 6.1 Objective

The objective was to maximize the total net income to be obtained from agricul~ tural production. The cotal net income was defined as the residual of the following transactions: receipts from selling water to the farmers, minus all costs of surface water and/or groundwater supply.

A "Varamin Water Supply Company" was to be established to operate and maintain the water distribution and supply facilities in the Plain. This company was to be incorporated into a "Varamin Development Authority" which, as a Government Service, conducts its business on a non~profit basis.

The term "maximization" was defined as the economic eriterion by which the marginal returns should be equal to or greater than the marginal costs. No restrictions were therefore defined where farmers have a fixed denand for water that must be supplied any cost. The demand for irrigation water in a certain (polygonal) sub-area was regarded as an agricultural production activity. in that area, and agricultural production was expressed in water quantities, instead of the more conmon way of expressing it as yield in kilograms per hectare.

The agricultural production activities in the various (polygonai) sub-areas. yielded different net returns per $\mathrm{m}^{3}$ of irrigation water and thus competed for the 1 imited available water resources.

### 6.2 Activities

In the linear programing model three main activities were distinguished:
i $\mathrm{PRD}=$ Agricultural production in polygon $i\left(\mathrm{~m}^{3}\right)$
$i$ WEL $=$ Supply of well water in polygon.i $\left(\mathrm{m}^{3}\right)$
i $S R F=$ Supply of surface water to polygon i $\left(a^{3}\right)$.

Each of these activities had costs: .

Cost of i PRD
The agricultural production in polygon $i$ was assumed to have a negative cost (=return). These negative costs represented the net return per $m^{3}$ of irrigation water and were based on:

1. an optimal cropping pattern for polygon i
2. the water requirements of this cropping pattern per hectare in polygon i
3. the net return per $m^{3}$ of irrigation water, which represented the net residual of the crop value after deduction of purchased input such as labour costs, interest, and capital costs of land levelling, if necessary. This net return, divided by the water demand, was regarded as the maximum price for water that farmers could afford to pay in polygon i.

Cost of i WEL
The $m^{3}$ price of well water was calculated for a standard type deep well. Since the depth to the water table and the aquifer transmissivity vary considerably throughout the basin, the energy costs to lift the groundwater to the surface vary too. Hence the standard $\mathrm{m}^{3}$ price of well water was adjusted for each polygonal area accordingly.

Cost of i SRF
The $\mathrm{m}^{3}$ price of surface water was calculated for an entirely new canal system capable of supplying each polygonal area up to its maximum water demand. The costs include the capital costs of surface water distribution, and the operation and maintenance of the canal system. It is obvious that when evaluating the linear programming solutions of water supply it may appear that the capacity of the canal system, or parts thereof, is too large. Under these circumstances the canal system should be redesigned and new cost values calculated. Such postoptimization amendments, however, were not made in this study.

### 6.3 Constraints

The above activities are subject to the following constraints:

$$
27
$$

b) $\quad \sum_{j=1}(\mathrm{i}$ SRF) $\leqslant$ MXRIV

This inequality states that the sum of the river water supplies to $j$ polygons must be equal to or less than the maximum quantity of river water available.

As explained in Chap. 2.6 , we had decided to use the following three river flows:

$$
150 \times 10^{6} \mathrm{~m}^{3}, \quad 220 \times 10^{6} \mathrm{~m}^{3}, \text { and } 340 \times 10^{6} \mathrm{~m}^{3} \text { a year }
$$

with return periods of respectively 20,5 , and 1.67 year. For MXRIV these three river flows were used.

$$
\sum_{j=1}^{27}(a \cdot j \text { WEL }) \leqslant \text { MXSFY }+\sum_{j=1}^{27}\left(b_{j} \cdot j \text { SRF }\right)
$$

This inequality states that the total net groundwater extraction by wells in $j$ polygons must be equal to or less than the maximum safe yield of the basin plus the total net deep percolation of river water in $j$ polygons. In this expression. a is the fraction of the pumped groundwater that is available at the field in polygon $j$ (or i) if $1 \mathrm{~m}^{3}$ of groundwater in polygon $j$ (or i) is recovered. It was assumed that there is 10 per cent return flow of the pumped groundwater, or $a=0.90$.

For convenience, though somewhat unconventional, the maximum safe yield of the basin (MXSFY) was defined as the difference between the total subsurface flow into the basin ( $65 \times 10^{6} \mathrm{~m}^{3}$ ) and the total subsurface flow from the basin ( $5 \times 10^{6} \mathrm{~m}^{3}$ ) or MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ per year.

The symbol $b_{j}$ stands for the total quantity of river water that percolates to the water table in polygon $j$, if $1 \mathrm{~m}^{3}$ of river water is released at the diversion weir at the apex of the alluvial fan to supply i polygons or

$$
b_{j}=\sum_{i=. .} e_{i j}
$$

where $e_{i j}$ is the portion of the river water that percolates to the water table in polygon i, if $1 \mathrm{~m}^{3}$ of river water is released at the diversion weir to supply polygon j . These percolation losses will be discussed in Chapter 8.4.

$$
\text { 3) } \quad i P R D \leqslant i M X D
$$

This inequality defines the maximum water demand restriction in polygon $i$. The maximum water demand of a polygonal area is based on:

- the hectares of land that can be cultivated *
- an optimum eropping pattern
- the water requirements of that cropping pattern
- the leaching requirements.

It is obvious that in determining the maximum water demand of the various polygonal areas we faced certain decision problems. What should be the future size of the farms? Should we supply all the irrigable land in the Plain or only the best land(Ciass I and II, see Fig.5)? Supplying only the best land would imply the resettlement of farmers from areas with poor quality soils to areas with better soils, which under the then prevailing system were only being partly used.

For the present study the above constraint was calculated for three different alternatives, which we have called Right Hand Side 1, 2, and 3. They are defined as follows:

RHS 1. This alternative assumes that irrigation water will only be supplied to the best lands (Class I and II), regardiess of the number of farmers living on these lands. The maximum water demand of polygon $i$ was determined by multiplying the hectares of Class I and Class II land in polygon $i$ by the specific water requirement coefficients, according to the optimal production pattern in this polygon.

RHS 2. This alternative assumes that irrigation water will be supplied to all farmers living on Class $I$ and II lands, each farmer possessing 3.85 ha , regardless of where these lands happen to be situated in the Plain. The maximum water demand of polygon $i$ (iMXD) was determined by multiplying the number of farmers in this polygon by 3.85 and by the specific water requirements of this polygon. For some polygons (l and 10 to l5) slightly different farm sizes were raken, namely 3.2 and 3.5 ha, because of the higher fertility of their soils.

RHS 3. This alternative is the same as RHS 2, except that a farm size of 6 ha was assumed. For the polygons $\mid$ and 22 the same farm size as in RHS 2 was assumed, while for polygons 10 to 14 farm sizes of respectively $5.4,5.4$, $4.1,4.7$, and 5.2 ha were taken.

Note that RHS 1 implies a policy of making a better use of the good soils in the Plain, whereas RHS 2 and 3 represent two possible development phases. With RHS 2 all available water resources will be used, whereas with RHS 3 it is assumed that, in addition, a quantity of water will be imported, thus allowing each farmer to cultivate 6 ha of land in the future.

$$
i \operatorname{DEM} \geqslant i \operatorname{PRD}-i \text { WEL }-c_{i} \cdot i \operatorname{SRF}
$$

This inequality defines the logical condition that for any level of agricultural production in polygon $i$, this polygon must be supplied with well water (WEL) or surface water (SRF).

The symbol $c_{i}$ stands for the fraction of the river water that is available at the field in polygon i, if a unit volume ( $\mathrm{m}^{3}$ ) is released from the diversion weir at the apex of the alluvial fan.

$$
\text { 5) } \quad \text { i MXSFY } \geqslant a \cdot i \text { WEL }-\sum_{j=\ldots} e_{i j} \cdot j \text { SRF }
$$

This inequality defines the maximum safe yield of polygon i. It states that the maximum safe yield of polygon $i$ must be equal to or greater than the difference between the net groundwater abstraction in polygon i and the total net deep percolation from canals in this polygon, if a unit volume ( $1 \mathrm{~m}^{3}$ ) of river water is released from the diversion weir to supply polygon $j$, or in other words, the accumulated canal percolation in polygon i when river water is passing this polygon to supply a downstream polygon $j$.

Remark

In the above notations of the constraints, the subscripts $i$ and $j$ were used to denote the number of polygons. Since there are 27 polygons, $i=1,2,3, \ldots 27$ and $j=1,2,3 \ldots 27$. For an array of numbers, as for instance in a linear programming matrix, the subscript $i$ usually refers to the row and the subscript $j$ to the column.

$$
27
$$

The sumation symbol $\sum_{j=1}$, in fact, does not refer to the polygons to 27 , but to the polygons $j=1$ to 23 inclusive and polygon 26 . In the linear. programming model, the polygons 24,25 , and 27 , which mainly cover the desert areas in the south of the Plain, were omitted due to the poor quality of their soils and groundwater.

For polygons 8 and 16 , only the activity river water supply was included in the matrix, because the high salinity of the groundwater in these areas prevents any substantial quantities of it being recovered.

Finally, the linear programing model was further simplified by omitting the activity of groundwater recovery by qanats. In a modern system of irrigation water supply these outdated, though ingeneous, facilities cannot be sufficiently relied upon to provide water at all times and in the quantities needed.

After having defined the various activities and constraints, we were able to draw up a linear programming matrix in the form shown in Table 29.

In this matrix the three activities agricultural production (PRD), well water supply (WEL), and surface water supply (SRF) appear in the row. Since there were 24 polygons in which these activities could be applied and since the polygons 8 and 16 had only two activities (PRD and SRF), the total number of activities was $3 \times 24-2=70$.

There is also acost row containing the unit volume costs of each activity. Note that for the activity PRD these costs are negative (=net return).

The constraints appear in the columns, starting with the resources: maximum river flows (MXRIV) and the maximum safe yield of the basin (MXSFY). Next in the columns are the three constraints of each polygon: maximum water demand (NXD), demand (DEM), and maximum safe yield (MXSFY). The model thus contains: $3 \times 24+2=74$ constraints.

As to the various coefficients in the matrix, it can be seen, for example, that performing the activity surface water supply (SRF) in polygon 1 at the level + 1 , will contribute to the maximum safe yield of the whole basin with a coefficient - $b_{1}$, due to the conveyance losses in the main canal and laterals. Further losses beyond the main canal and laterals result in an availability at the field of $-c_{f}$. Any unit volume of river water passing polygon $l$ to supply downstream polygons will contribute to the maximum safe yield of polygon 1 with a coefficient - $e_{1]}$.

In the next columns, which refer to the activity SRF, the same arrangement of the coefficients appears, except that river water supplied to, for instance, polygon 2 must pass through polygon 1 . This activity contributes not only to the maximum safe yield of polygon 2 by a coefficient $-e_{22}$, but also to that

TABLE 29. Scheme of the linear programming matrix

| Polygon No. | 01 | 01 | , 01 | 02 | 02 | 02 | 03 | 03 | 03 | 04 | 04 | 04 | 05 | 05 | 05 | 06 | 06 | 06 | etc. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Activity | PRD | WEL | SRF | PRD | WEL | SRF | PRD | WEL | SRF | PRD | WEL | SRF | PRD | WEL | SRF | PRD | WEL | SRF |  |
| Cost Row | -1 | +1 | +1 | -1 | $+1$ | +1 | -1 | +1 | +1 | -1 | +1 | $+1$ | -1 | $+1$ | $+1$ | -1 | $+1$ | $+1$ |  |
| MXRIV |  |  | +1 |  |  | +1 |  |  | +1 |  |  | +1 |  |  | +1 |  |  | +1 |  |
| MXSFY |  | +a | -b, |  | *a | $-b_{2}$ |  | +a | $-\mathrm{b}_{3}$ |  | +a | $-b_{4}$ |  | + | $-b_{5}$ |  | + ${ }^{\text {a }}$ | $-b_{6}$ |  |

Polygonal comstraints

of polygon 1 by a coefficient $-e_{12}$, see also Fig. 29 .
Once the general linear programing matrix had been drawn up,there remained the matter of preparing the cost values and the values of the coefficients.This will . be the subject of the next chapter, which will also shed some further light on the procedures discussed above.

## 7. Calculating costs of activities

The three activities of the linear programming model - agricultural production, river water supply, and groundwater supply - all have costs, and thesediffer from one polygon to another. How we calculated the cost values of these activities will now be explained. It must be emphasized that the basic data and prices used refer to the years 1968/1969.

### 7.1 Polygonal costs of agricultural production

In calculating the (negative) cost of agricultural production or net return per $\mathrm{m}^{3}$ of irrigation water in each polygon, we made use of: a cropping pattern, the hectares to be cultivated, the value of output, the value of input, and the water demand of each crop.

In studies of this kind it. is common practice for the subsystems of the agricultural production system to be optimized. Although linear programming is a technique that allows optimum cropping patterns to be found, the specific data required for this purpose were not available. More information was acquired as the studies progressed, but it was still not sufficient for us to apply linear programming. We therefore used a preliminary cropping pattern developed by the project's agronomist. He had divided the Plain into five zones, A, B, C, D, and $E$, and for each zone had developed a cropping pattern which, under the given circumstances, could be considered "the best".

These cropping patterns are presented in Fig. 27.

For the calculation of the weighted net return per $m^{3}$ of water for the five cropping patterns, we refer to Tables 21 and 22 . As will be seen later, the values of net return per $\mathrm{m}^{3}$ of water for each polygon (but with a negative sign) were entered in the cost row of the matrix under PRD (see Table 45).

### 7.2 Polygon cost of surface water

As a basis for the calculation of the $m^{3}$ cost of surface water in each polygon, a canal system, capable of supplying all the 57,487 ha of Class I and Class II land with $11,500 \mathrm{~m}^{3}$, of water per hectare per year, was designed. In principle, this meant that the entire water demand of that area could be met by supplying surface water only, provided that sufficient quantities of surface water were available.

It was assumed that the $\mathrm{m}^{3}$ cost of surface water would not change substantially if another canal layout were employed: the higher $\mathrm{m}^{3}$ cost of water supplied by a smaller capacity canal system would be offset by the lower capital cost of the canal system. This assumption does not seem too critical as can be seen from the following comparison between cost figures for different canal layouts obtained from preliminary cost calculations (Table 30).

TABLE 30. Cubic metre cost of surface water for different canal layouts

| Gross ha <br> supplied | Net ha <br> supplied | Annually <br> supplied $\mathbf{m}^{3}$ | Investaent <br> in Rials | Rials <br> per $m^{3}$ |
| :--- | :--- | :---: | :--- | :--- |
| 68,100 | 61,290 | $704 ; 835,000$ | $977,100,000$ | 1.386 |
| 53,100 | 47,790 | $549,585,000$ | $667,300,000$ | 1.214 |
| 33,480 | 30,132 | $346,518,000$ | $428,788,000$ | 1,237 |

It was further assumed that the proportions of polygonal $m^{3}$ costs of surface water calculated for this maximum supply system would apply equally well to any canal layout suggested by the water supply solution generated by the model.

The canal system used in the calculations is presented in a simplified form in Fig. 29. Only the main canals and laterals are indicated, since showing the numerous sublaterals and field ditches would have necessitated a much larger map.

The costs of land levelling were not included in the calculations, these being allocated to the costs at farm level.

The cost items that were taken into account and their values, converted to annual costs, are shown in Table 31.


Fig.29. Simplified conal lay-out, showing main canals and laterals, and the polygon network.

TABLE 31. Cost items used in calculating the $\mathrm{m}^{3}$ cost of surface water

| Item | Interest | Operation * Maintenance | Depreciation | Annually |
| :---: | :---: | :---: | :---: | :---: |
|  | \% of investment |  |  |  |
| Diversion weir | 6 | 2 | 2 | 10 |
| Drain ditches | 6 | 2 | 2 | 10 |
| Storm \& flood control | 6 | 2 | 2 | 10 |
| Gravel roads | 6 | 2 | 2 | 10 |
| Dirt roads | 6 | 2 | 2 | 10 |
| Farm diches | 6 | 2 | 2 | 10 |
| Concrete-ifined canals | 6 | 2 | 2 | 10 |
| Lined canals | 6 | 2 | 3.33 | 11.33 |

For certain of the components listed in Table 31, the total annual cost was allocated to the sub-areas in proportion to the quantity of water they will receive. The procedure of allocating these costs was as follows. Each main canal was divided into segments $S_{1}, S_{2}, S_{3}$, etc. A segment is that stretch of the canal between two off-takes (see Fig.30). Each segment carries the flow $Q S$, which is composed of the fractional flows $q$ supplying the downstream sub-areas fed by this segment. The first-segment $S_{\text {, }}$ carries all the fractional flows $q_{A}, q_{B}, q_{C}$, $q_{D}, q_{E}$, and $q_{F}$ supplying the sub-areas $A, B, C, D, E$, and $F$. The last segment $S_{3}$ carries only the fractional flows $q_{D}, q_{E}$, and $q_{F}$ supplying the sub-areas D, E, and F.

The annual costs of the segments, $C S_{1}, C S_{2}, C S_{3}$, etc., were calculated and then allocated in the following way. For illustration, let us take the segment $S_{3}$, carrying the flow $Q S_{3}$ and supplying the sub-areas $D, E$, and $F$.

The cost allocated to sub-area $D$ was $\left(q_{D} / Q S_{3}\right) \mathrm{C}_{3}$, and to sub-area $E,\left(q_{E} / Q S_{3}\right)$ $\mathrm{C}_{3}$, and to sub-area $\mathrm{F},\left(\mathrm{q}_{\mathrm{F}} / \mathrm{Q} \mathrm{S}_{3}\right) \mathrm{C}, \mathrm{S}_{3}$.

The criterion for the division into sub-areas was a discrete set of laterals. The annual cost of a set of laterals making up a sub-area was allocated to that sub-area.

The cotal annual cost allocated to a sub-area was divided by the quantity of water supplied to this sub-area annually; in this way we obtained the annual cost of one $m^{3}$ of surface water.


Eig. 30: Scheme of a main canal and laterals, which supply surface water to subareas $A, B, C, D, E$, and $F$.

Finally the polygonal network was superimposed on the canal lay-out to determine the polygonal $\mathrm{m}^{3}$ cost of surface water. Obvious 1 y , canals from different hydraulic sub-areas were found to fall within a particular polygon.

The areal proportions of the different sub-areas in all the polygons were determined, after which the polygonal $\mathrm{m}^{3}$ cost of surface water was calculated from the sub-area $\mathrm{m}^{3}$ cost, weighted for the above proportions.

For a better understanding of the above procedure the reader may appreciate the following numerical example. Let us choose polygon 12. This polygon forms part of a hydraulic sub-unit made up mainly of areas belonging to the polygons 4,11 , and 12 and is therefore designated "cost area $4 / 11 / 12$ ". This area will be supplied by one of the two main canals, indicated as main canal V . This canal is composed
of the segments $V-S_{1}, V-S_{2}, V-S_{3}$, etc.
The cost of the segments was calculated and converted to annual costs by multiplying by 0.10 (see Table 31). A quantity of $49,197,000 \mathrm{~m}^{3}$ surface water wili be supplied to this cost area if the demand of $11,500 \mathrm{~m}^{3} /$ ha is supplied to the maximum available land of Class I and Class II. Table 32 shows the cost allocation data for main canal $V$.
It can be seen from this table that canal $V$ carries $543,951,000 \mathrm{~m}^{3}$ a year, of which $49,197,000 \mathrm{~m}^{3}$, or 9 per cent, is supplied to cost area 4/11/12.

Since the total cost of the first segment of this main canal (V $\quad$ S.l) is Rials $8,200,000$, we allocated a cost of $0.09 \times \mathrm{Rls.8,200,000}=\mathrm{Rl} \mathrm{s} .738,000$ a year to cost area $4 / 11 / 12$.
The segments $S_{2}$ and $S_{3}$ of this main canal also supply cost area $4 / 11 / 12$. Hence, for segment $V-S_{2}$ we allocated a cost of $0.317 \times \mathrm{R} 1 \mathrm{~s}, 2,900,000=919,300 \mathrm{Rls}$ a year to cost area $4 / 11 / 12$ and for segment $V-S_{3}$ a cost of $0.319 \times$ R1s. $5,400,000=1,722,600 \mathrm{Rls}$ a year.

The total cost for the main canal allocated to cost area $4 / 11 / 12$ is therefore 3,379,900 R1s a year.

(TABLe 32, cont.)

| Main canal segment (cont.) |  |  |  | v.2-s.2 |  |  | v. $2-5.3$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| cost area | $\begin{aligned} & \text { tillion } \mathrm{m}^{3} \text {. } \\ & \text { per year } \end{aligned}$ | 2 of yearly flow | Cost per ycar <br> (R1s) | $\begin{aligned} & \text { million m } \\ & \text { per year } \end{aligned}$ | 2 of yearly flow | Cost per year (R15) | $\begin{aligned} & \text { Million m } \\ & \text { per year } \end{aligned}$ | x of yearly flow | Cost per year <br> (R1s) | Total cost of main canal $V$ Rls/year |
| 7 | 8.936 | 5.2 | 232,180 |  |  |  |  |  |  | 363,380 |
| 9/10/5 | 34.707 | 20.3 | 906, 395 | 34.707 | 21.5 | 1.786.650 |  |  |  | 3.217,845 |
| 15-18 | 127.005 | 74.5 | 3,326,425 | 127.005 | 78.5 | 6,523,350 | 127.005 | 100 | 3,195,000 | 14,963,575 |
| 5/10/11 |  |  |  |  |  |  |  |  |  | 3,212,000 |
| 4/11/12 |  |  |  |  |  |  |  |  |  | 3,379,900 |
| 13/14/19 |  |  |  |  |  |  |  |  |  | 5,909,500 |
| $2 / 3$ |  |  |  |  |  |  |  |  |  | 2,758,160 |
| 3 |  |  |  |  |  |  |  |  |  | 1,384,770 |
| 9-23/26 |  |  |  |  |  |  |  |  |  | 13,128,870 |
| TOTAL | 170.648 | 100 | 4.465,000 | 161.712 | 100 | 8,310,000 | 127.005 | 100 | 3,195,000 | 48,318,000 |

Table 33 shows the investment costs of canals beyond the main canal (laterals, tertiaries, and lined quaternaries) in cost area 4/11/12.

TABLE 33. Sumnary of costs of canals beyond the main canals for cost area $4 / 11 / 12$

| Main canal unit | Class of canal | Investment <br> (million R1s) | - . |
| :---: | :---: | :---: | :---: |
| v-3 | 1ateral | 2.482 |  |
| v-3-1 | tertiary | 2.270 |  |
|  | lined quaternary | 3.500 |  |
| v-5 | lateral | 2.901 |  |
|  | terciary | 2.755 |  |
| - | lined quaternary | 3.840 |  |
| v-7 | lateral | 3.320 |  |
|  | tertiary | 2.270 |  |
|  | lined quaternary | 4.200 |  |
| v-9 | lateral | 6.800 |  |
|  | tertiary | 5.500 |  |
|  | lined quaternary | 8.000 | . |
|  | total | 48.338 |  |
|  | anmual cost | $48.338 \times 0.11$ | - R16. 5,476,695 |

The yearly percentage of investment for a canal system with ined quaternaries is 11.33 (see Table 31). Hence the total investment for this cost area must be multiplied by 0.1133 to find the annual cost of laterals, tertiaries, and lined quaternaries in cost area 4/11/12.

Table 34 shows the costs of the canal system components, allocated to the various cost areas on a per hectare basis.

The annual cost of these components was taken at 10 per cent of the total investment (see Table 31). Hence, $0.10 \times$ Rls. $9,600=$ Rls. $960 /$ ha a year.

Cost area $4 / 1 \mathrm{t} / 12$ has a net agricultural production area (Class 1 and II land of 4,782 ha, so chat for the above components a cost of $4,782 \times 960=4,106,880$ R1s/year must be allocated to that area.

TABLE 34. Costs of canal system components allocated to the various cost areas on a per hectare basis

| Component | Life time (years) | R1s per net ha |
| :---: | :---: | :---: |
| Diversion structure (R1s $35 \times 10^{6}$ ) | 50 | 600 |
| Drainage ditches ( $40 \mathrm{~m}^{3} / \mathrm{ha}$ ) | 50 | 3,000 |
| Stotyp \& flood control ditches ( $15 \mathrm{~m}^{3} / \mathrm{ha}$ ) | 50 | 1,000 |
| Gravel roads ( $6 \mathrm{~m} / \mathrm{ha}$ ) | 50 | 1,000 |
| Dirt roads ( $40 \mathrm{~m} / \mathrm{ha}$ ) | 50 | 1,000 |
| Farm ditches | So | 3,000 |
| TOTAL |  | 9,600 |

The total annual costs of surface water in area $4 / 11 / 12$ is therefore the sum of the following items:

| Main canal | $3,379,900$ | $\mathrm{Rls} / \mathrm{year}$ |
| :--- | ---: | :--- |
| Lacerals, tertiaries, and lined quaternaries. | $5,476,695$ | $\mathrm{R} 1 \mathrm{~s} / \mathrm{year}$ |
| Canal system components | $4,106,580$ | $\mathrm{R} 1 \mathrm{~s} / \mathrm{year}$ |
| TOTAL | $12,963,475$ | $\mathrm{Rls} / \mathrm{year}$ |

As discussed earlier, the maximum possible water demand of area $4 / 11 / 12$ is $49,197,000 \mathrm{~m}^{3} /$ year. Hence, if we divide the total annual cost of surface water in the area by that water quantity, we find the $\mathrm{m}^{3}$ cost of surface water to be $12,963,475: 49,197,000=0.2635 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$.

In a similar way the $m^{3}$ cost of surface water was calculated for all the other cost areas.

The final step was the calculation of the polygonal $\mathrm{m}^{3}$ cost of surface water. For this purpose the polygonal network was superimposed on the map of the canal layout. Obviously, the boundaries of the cost areas did not coincide with those of the polygonal areas, being based on entirely different criteria. It was found that a particular polygon covered minor or major parts of two or more cost areas. Hence the area of each portion of the cost areas falling within the boundaries of a particular polygon was measured by planimeter. Since the surface area of the polygon was known, the area percentage of each cost area occurring within the polygon could be calculated.

In the example of polygon 12, it was found that of the total polygonal area 96.3 per cent was covered by cost area $4 / 11 / 12$ and 3.7 per cent by cost area 3 , for
which a surface water cost of $0.227 \mathrm{IRls} / \mathrm{m}^{3}$ had been calculated. The cost of surface water in polygon 12 was therefore:

$$
(0.2635 \times 0.963)+(0.2271 \times 0.037)=0.2621 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}
$$

The $m^{3}$ cost of surface water for all the other polygons was calculated in the same' way. Because of space limitations we shall not present the long tables and detailed calculations required for that purpose and shall assume that the above example suffices to explain how the various polygonal $\mathrm{m}^{3}$ costs were calculated. Table 35 shows the results of our calculations. These figures were entered in the linear programing matrix under the column SRF of each polygon (see Table 45).

TABLE 35. Cubic metre cost of surface water in the different polygons

| Polygon No. | Cost $\left(\mathrm{Els} / \mathrm{ma}^{3}\right)$ | Polygon No. | Cost (Rls/mi) |
| :---: | :---: | :---: | :---: |
| 1 | 0.1833 | 13 | 0.3193 |
| 2 | 0.3058 | 14 | 0.2926 |
| 3 | 0.2814 | 15 | 0.3016 |
| 4 | 0.2567 | 16 | 0.3167 |
| 5 | 0.2599 | 17 | 0.3167 |
| 6 | 0.2420 | 18 | 0.3172 |
| 7 | 0.2904 | 19 | 0.3241 |
| 8 | 0.3338 | 20 | 0.3555 |
| 9 | 0.2542 | 21 | 0.3555 |
| 10 | 0.2548 | 22 | 0.3555 |
| 11 | 0.2426 | 26 | 0.3555 |
| 12 | 0.2621 |  |  |

No surface water will be supplied to polygon 23 because of the high costs; nor will any be supplied to polygons 24,25 , and 27 , which are desert areas. Figure 31 shows the polygonal distribution of surface water cost per $\mathrm{m}^{3}$.


Fig. 31. Cost of surface water per $m^{3}$ and per polygon.

### 7.3 Polygonal cost of well water

The $m^{3}$ cost of well water was calculated for a standard type well, $12^{\prime \prime}$ in diameter, 150 m deep, and yielding $200 \mathrm{~m}^{3}$ per hour. Since the depth to the water table and the aquifer transmissivity vary considerably throughout the basin, the $m^{3}$ price of well water was adjusted accordingly. The polygonal well water prices were calculated on the basis of the safe yield concept, i.e. for a relatively stable regional water table. No well water prices were calculated for a water table considerably deeper than the present one, i.e. no mining of the groundwater resource was considered. The linear programming model used did not allow cost values to vary during the computations.

Cost of a model well

For a $12^{\prime \prime}$ well, 150 m deep, and yielding $200 \mathrm{~m}^{3}$ per hour, the following cost items and cost figures were taken into account:

| Cost of drilling: $1,200 \mathrm{R} 1 \mathrm{~s}$ per metre | R1s. | 180,000 |
| :--- | ---: | ---: |
| Cost of casing and well screen | 195,000 |  |
| Cost of development of the well ( 24 hours) | 35,000 |  |
| Cost of pump to be installed at 70 m depth | 380,000 |  |
| Cost of Diesel engine, 116 hp | 415,000 |  |
| Cost of protection building | 150,000 |  |
| Cost of mobilization of drilling rig, etc. | 27,000 |  |

TOTAL INVESTMENT
R16. 1,382,000

## Effective time of operation

Diesel-engine driven pumps require maintenance and it was therefore assumed that the engine would operate for 22 hours a day. The total monthly production would therefore be $30 \times 22 \times 200=132,000 \mathrm{~m}^{3}$, which corresponds to 25 per cent of the total water demand.

To meet the total demand, the well should operate for 3000 hours, which corresponds to a total production of $3000 \times 200=600,000 \mathrm{~m}^{3}$.

Operating cost
3,000 hours at a cost of $69.9 \mathrm{Rls} /$ hour
$(0.6 \mathrm{Rls} / \mathrm{hp})$
Maintenance and operator cost
( $2.5 \%$ of investment)
34,550
rotal
Rls. 243,350

Depreciation

| Well, assumed life time 25 years | R1s. | 17,480 |
| :---: | :---: | :---: |
| Equipment, engine, pump, assumed life time 12 years |  | 65,833 |
| Protection house, assumed life time 30 years |  | 25,000 |
| total | R1s. | 108,313 |

Interest charges on investment
6 per cent of the investment (Rls.1, 382,000 ) Rls. 82,920

Total annual cost

| Operating cost |  | Rls. | 243,350 |
| :--- | :--- | :--- | ---: |
| Depreciation |  |  | 108,313 |
| Interest |  |  | 82,920 |
| TOTAL |  | R1s. | 434,583 |

Cost of well water

| Annual cost | R1s. | 434,583 |  |
| :--- | ---: | ---: | ---: |
| Annual extraction | $600,000 \mathrm{~m}^{3}$ |  |  |
| Cost of well water | $434,583 / 600,000=$ | R1s. | $0.72 / \mathrm{m}^{3}$ |

Required horse power

The required horse power of the pump engine was found from the formula

$$
\begin{aligned}
\mathrm{hp} & =\frac{\text { well discharge } \times \text { water 1ift }}{270 \times \text { pump efficiency }} . \\
& =\frac{200 \times 70}{270 \times 0.50}=104+10 \%=116 \mathrm{hp}
\end{aligned}
$$

Although the usual pump efficiency is taken to be $75 \%$, the Varamin Plain lies $1,000 \mathrm{~m}$ above sea level and has extremely high temperatures in sumer, so that an efficiency of 50 per cent was assumed.

## Calculating the cost of well water per polygon

One of the items that determines the price of a well is the height over which the water must be lifted to reach the surface. The higher the lift, the more energy is required, and the more powerful must be the engine. Diesel engines are commonly used in the Plain, because electricity is not (yet) available.

The depth at which the pump must be installed in a well depends primarily on the depth to the water table. Referring back to Fig. 24 , it can be seen that the depth to the water table varies from less than 5 m in the lower parts of the Plain to more than 80 m in the upper part. To estimate the average depth to the water table in each polygon, the polygon network was superimposed on the map of Fig. 24.

A well screen must be installed in the saturated part of the aquifer. Assuming certain well losses and a safety margin to ensure that the well screen will not be pumped dry, it follows that the well must be drilled to at least $25, \mathrm{~m}$ below the water table. Any groundwater pumpage will cause the water table to drop. The drop will be steep if the well is drawing from an aquifer of low transmissivity, and only slight if the transmissivity is high, assuming the well yields are the same. The transmissivity must therefore be taken into account in calculating the depth of the wells in the various polygons. Based on an exami-
nation of the yields of the existing wells in the Plain, we assumed that a well in any polygon would yield, on the average, $200 \mathrm{~m}^{3} / \mathrm{hr}$ or $56 \mathrm{l} / \mathrm{s}$. The question then arose, what would be the drawdown for a well of such capacity in the various polygons? The answer could only be very approximate, as adequate data were not available. It has long been known that the transmissivity of an aquifer equals approximately the specific capacity of a well drawing from this aquifer. (The specific capacity of a well is its yield per metre drawdown in the well, or $T \simeq 1.22 \mathrm{Q} / \mathrm{s})$.

Using this approximation and estimating the average transmissivity of each polygon from Fig. 21, we estimated the drawdown of a well yielding $200 \mathrm{~m} / \mathrm{hr}$ for each polygon. This drawdown was added to the estimated well depth to find the ultimate depth at which the pump should be installed.

A glance at Fig. 21 shows that the aquifer transmissivity varies considerably throughout the basin and even within a single polygon. Hence the assumed average transmissivity values are rough estimates only; they may be higher or lower, depending on the site selected in a polygon.

After estimating the depth at which the pumps were to be installed, we determined the required horse power of the engine, using the formula presented above. The price of the engines was taken from catalogues.

The results of the above calculations are summarized in Table 36.

Applying the same procedure as for the standard type well, we obtained the cost of well water in each polygon. The results of these calculations are shown in Table 37 and in Fig. 32.

TABLE 36. Pump engine capacities and their costs per polygon

| Polygon number | Max.well depth | Transmissivity | Drawdown | Water <br> table <br> depth | Pumping depth | $\frac{\text { Horse }}{\text { required }}$ | $\frac{\text { power }}{\text { available }}$ | Cost of engine |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (to) | $\left(\mathrm{m}^{2} /\right.$ day $)$ | (m) | (m) | (m) |  | . | (1,000 R1s) |
| 1 | 325 | 5,000 | 2.1 | 93 | 121.1 | 179.3 | 188 | 680 |
| 2 | 340 | 4,800 | 2.3 | 49 | 77.3 | 114.5 | 116 | 415 |
| 3 | 265 | 2,000 | 3.1 | 15 | 44.1 | 65.3 | 60 | 340 |
| 4 | 290 | 5,000 | 2.1 | 37 | . 65.3 | 96.7 | 116 | 415 . |
| 5 | 315 | 5,000 | 2.1 | 38 | 66.4 | 98.3 . | 116 | 415 |
| 6 | 200 | 4,000 | 2.6 | 39 | 67.6 | 113.4 | 116 | 415 |
| 7 | 110 | 1,500 | 4.8 | 17 | 47.8 | 70.8 | 116 | 415 |
| 8 | 100 | 150 | 40.0 | 25 | 91.0 | 134.7 | 135 | 520 |
| 9 | 115 | 1,100 | 4.6 | 19 | 49.6 | 73.5 | 116 | 415 |
| 10 | 195 | 1,500 | 4.0 | 14 | 44.0 | 65.2 | 60 | 340 |
| 11 | 268 | 2,000 | 3.1 | 20 | 49.1 | 72.7 | 116 | 415 |
| 12 | 285 | 2,000 | 3.1 | 15 | 44.1 | . 65.3 | 60 | 340 |
| 13 | 365 | 1;600 | 3.8 | 19 | 48.8 | 72.3 | 116 | . 415 |
| 14 | 185 | 1,000 | 5.3 | 21 | 52.3 | 77.5 | 116 | 415 |
| 15 | 290 | 600 | 9.1 | 26 | 61.1 | . 90.5 | - 116 | 415 |
| 16 | 262 | 150 | 40.0 | 12 | 78.0 | 115.5 | 116 | 415 |
| 17 | 122 | 250 | 22.0 | 14 | 66.0 | 97.7 | 116 | 415 |
| 18 | 185 | 550 | - 11.1 | 19 | S6. 1 | 83.1 | 116 | 415 |
| 19 | 245 | 1,100 | 5.3 | - 19 | 50.3 | 74.5 | 116 | 415 |
| 20 | 270 | 1,600 | 3.8 | 23 | 52.8 | 78.2 | 116 | 415 |
| 21 | 225 | 1,400 | 4.3 | 15 | 45.3 | 67.1 | 60 | 340 |
| 22 | 265 | 1,000 | 5.7 | 13 | 44.7 | 66.2 | 60 | 340 |
| 23 | 190 | 300 | 18.2 | 7 | 51.2 | 75.8 | 116 | 415 |
| 26 | 200 | 350 | -16.8 | 16 | 58.5 | 87.1 | 116 | 415 |

TARLE 37. Cost calculations of well water in the different polygons (assumed annual extraction for each well: $600,000 \mathrm{~m}^{\mathrm{3}}$ )

| Polygon No. | Cose ( $\times 1,000$ RLs) |  |  |  | Total investment (col. 1+24 3+4)(x),000R1s.) | Cost |  | $\begin{aligned} & \text { Incerest } \\ & 6 \% \end{aligned}$ | Depreciation |  |  |  | Total depreciation $(9+10+11+12)$ | $\begin{aligned} & \text { Total } \\ & \text { annusal } \\ & \text { costs } \\ & (6+7+ \\ & 8+3) \\ & (\times 1,000 \\ & \text { R1s. }) \end{aligned}$ | $\mathrm{Rls} / \mathrm{m}^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | pump | drilling | $\begin{aligned} & \text { casing } \\ & \text { screen } \end{aligned}$ | building, moving of rig |  | opera- <br> cion | $\begin{gathered} \text { cmaiń- } \\ \text { tenamee } \end{gathered}$ |  | building $s$ wove of rig. | engins | pump | $\begin{aligned} & \text { casing } \\ & \text { sdril- } \\ & \text { ling } \end{aligned}$ |  |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | is |
| 1 | 600 | 145.3 | 157.4 | 212.0 | 1,815,9 | 322.7 | 45.4 | 108.9 | 9.5 | 56.7 | 50.0 | 12.1 | 127.3 | 604.3 | 1.0073 |
| 2 | S20 | 92.8 | 100.5 | 212.0 | 1,340,3 | 206.1 | 33.5 | 80.4 | 8.5 | 34.6 | 43.3 | 7.7 | 94.1 | 414.1 | 0.6901 |
| 3 | 360 | 52.9 | 57.3 | 212.0 | 1,022.2 | 117.5 | 25.6 | 61.3 | 8.5 | 28.3 | 30.0 | 4.4 | 71.2 | 275.6 | 0.4593 |
| 4 | 450 | 78.4 | 64,9 | 212.0 | 1,240.3 | 174.1 | 31.0 | 74.4 | 8.5 | 34.6 | 37.5 | 6.5 | 87.1 | 366.6 | 0.6110 |
| 5 | 450 | 79.7 | 86.3 | 212.0 | 1,243.0 | 176.9 | 31.1 | 74.6 | 8.5 | 34.6 | 37.5 | 6.6 | 87.2 | 369.8 | 0.6163 |
| 6 | 450 | 81.1 | 87.9 | 212.0 | 1,246.0 | 204.1 | 31.1 | 74.8 | 8.5 | 34.6 | 37.5 | 6.8 | 87.4 | 397.4 | 0.6623 |
| 7 | 360 | 37.4 | 62.1 | 212,0 | 1,106.5 | 127.4 | 27.7 | 66.4 | 8.5 | 34.6 | 30.0 | 4.8 | 77.9 | 299.5 | 0.4990 |
| 8 | 550 | 109.2 | 118.3 | 212.0 | 1,509.5 | 242.5 | 37.7 | 90.6 | 8.5 | 43.3 | 45.8 | 9.1 | 106.7 | 477.5 | 0.7958 |
| - 9 | 360 | 59.5 | 64.5 | 212.0 | $1,111.0$ | 132.3 | 27.8 | 66.7 | 8.5 | 34.6 | 30.0 | 5.0 | 78.1 | 304.9 | 0.5082 |
| 10 | 360 | \$2.8 | 57.2 | 212.0 | 1,022.0 | 117.4 | 25.6 | 64.3 | 8.5 | 28.3 | 30.0 | 4.4 | 71.2 | 275.5 | 0.4592 |
| 11 | 360 | 58.9 | 63.9 | 212.0 | 1,109.7 | 130.9 | 27.3 | 66.6 | 8.5 | 34.6 | 30.0 | 4.9 | 78.0 | 303.2 | 0.5053 |
| 12 | 360 | 52.9 | 57,3 | 212.0 | 1,022,2. | 137.5 | 25.6 | 61.3 | 8.5 | 28.3 | 30.0 | 4.4 | 71.2 | 275.6 | 0.4593 |
| 13 | 360 | \$8.6 | 63.4 | 212.0 | 1,109.0 | 130.1 | 27.7 | 66.5 | 8.5 | 34.6 | 30.0 | 4.9 | 78.0 | 302.3 | 0.5038 |
| 14 | 360 | 62.8 | 68.0 | 212.0 | 1,117.8 | 139.5 | 27.9 | 67.1 | 8.5 | 34.5 | 30.0 | 5.2 | 78.3 | 312.8 | 0.5213 |
| 15 | 450 | 73.3 | 79.4 | 212.0 | 1.229 .7 | 162.9 | 30.7 | 73.8 | 8.5 | 34.6 | 37.5 | 6.1 | 36.7 | 354.1 | 0.5902 |
| 16 | 520 | 93.6 | 101.4 | 212.0 | 1,342.0 | 207.9 | 33.5 | 80.5 | 8.5 | 34.6 | 43.3 | 7.8 | 94.2 | 416.1 | 0.6935 |
| 17 | 430 | 79.2 | 85.8 | 212.0 | 1,242.0 | 175.9 | 31.0 | 74.5 | 8.5 | 34.6 | 37.5 | 6.6 | 87.2 | 368.6 | 0.6143 |
| 18 | 450 | 67.3 | 72.3 | 212.0 | 1,216.6 | 149.6 | 30.4 | 73.0 | 8.5 | - 34.6 | 37.5 | 5.6 | 86.2 | 339.2 | 0.5653 |
| 19 | 360 | 60.4 | 65.4 | 212.0 | 1,112.8 | 134.1 | 27.8 | 66.8 | 8.5 | 34.6 | 30.0 | 5.0 | 78.1 | 306.8 | 0.5113 |
| 20 | 360 | 63.4 | 68.6 | 212.0 | 1,119.0 | 140.8 | 28.0 | 67.1 | 8.5 | 34.6 | 30.0 | 5.3 | 78.4 | 314.3 | 0.5238 |
| 21 | 360 | 54.4 | 58.9 | 212.0 | 1,025.3 | 120.8 | 25.6 | 61.5 | 8.5 | 28.3 ${ }^{\circ}$ | 30.0 | 4.5 | 31.3 | 279.2 | 0.4653 |
| 22 | 360 | 53.6 | 58.1 | 212.0 | 1,023.7 | 119.2 | 25.6 | 61.4 | 8.5 | 28.3 | 30.0 | 4.5 | 71.3 | 277.5 | 0.4625 |
| 23 | 360 | 61.4 | 66.6 | 212.0 | 1,115.0 | 136.4 | 27.9 | 66.9 | 8.5 | 34.6 | 30.0 | 5.1 | 78.2 | 309.4 | 0.5157 |
| 26 | 430 | 70.6 | 76.4 | 232.0 | 1,224.0. | 156.8 | 30.6 | 73.4 | 8.5 | 34.6 | 37.5 | 5.9 | 86.5 | 347.3 | 0.5788 |



Fig.32. Cost of wetl water per $m^{3}$ per polygon.

## 8. Calculating resource constraints and coefficients

### 8.1 Surface water constraints

Since it is unlikely that another storage dam will be,or even can be, constructed in the Jaj Rud, we had to regard the Jaj Rud flow as a stochastic problem. As was explained in Chap. 2, Section 6 , we assumed the following surface water flows available to the Plain:

$$
\begin{aligned}
& \text { MXRIV }=150 \times 10^{6} \mathrm{~m}^{3} \text { a year, with a return period of } 20 \text { years } \\
& \text { MXRIV }=220 \times 10^{6} \mathrm{~m}^{3} \text { a year, with a return period of } 5 \text { years } \\
& \text { MXRLV }=340 \times 10^{6} \mathrm{~m}^{3} \text { a year, with a return period of } 1.67 \text { year }
\end{aligned}
$$

It is recalled that these flows include the $80 \times 10^{6} \mathrm{~m}^{3}$ annually diverted to Teheran. It was assumed that this quantity would be replaced in one way or another, because otherwise agriculture in the Plain would suffer, and part of its population would be deprived of their only source of income.

The above surface water flow constraints were entered in the matrix, but it will be obvious that they can easily be replaced by other, more updated constraints, if the need should arise.

### 8.2 Groundwater constraints

The quantity of groundwater that can be withdrawn annually from the basin, the maximum "safe yield" (MXSFY), was set at $60 \times 10^{6} \mathrm{~m}^{3}$, being the difference between the total subsurface inflow and the total subsurface outflow.

### 8.3 Maximum water demand of the polygons

In Chap. 6, Section 3, we defined this constraint and indicated three alternative policies: RHS 1, RHS 2, and RHS 3. For each of these alternatives, we calculated the maximum water demand for each polygon. The calculations were based on the hectares of Class I and Class II land in the polygon (Fig.5), the number of farmers living in the polygon, and the water demand in in ${ }^{3}$ per hectare for the accepted cropping pattern of the polygon. The results are shown in Table 38.

TABLE.38. Calculation of maximum wáter demand per polygon for RHS1, RHS2, and RHS3

| Polygon <br> No. | Area of polygon | Number of farmers | Average water demand | Maximum water demand (in million m ${ }^{3}$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Area of irrigable <br> land. Cl as- <br> ses I \& II | RHS ! <br> iMXD | Area of irrigable land | RHS' 2 <br> imXD | Area of irrigable land. | RHS 3 <br> IMXD |
|  | (ha) | - | $\left(m^{3} / \mathrm{ha}\right)$ | (ha) |  | (ha) ${ }^{\text {2 }}$ | $\cdot$ | (ha) ${ }^{2}$ |  |
| I | 3,857.5 | 106 | 12,012 | 338 | 4.060 | 338 | 4.060 | . 338 | 4.060 |
| 2 | 6,850.0 | 469 | 10,827 | 3,127 | 33.856 | 1,806 | 19.554 | 2,814 | 30.467 |
| 3 | 5,337.5 | 493 | 10,827 | 4,590 | 49.696 | 1,898 | 20.550 | 2,958 | 32.026 |
| 4 | 3,590.0 | 93 | 11,196 | 1,260 | 14:099 | 358 | 4.008 | 358 | 6.247 |
| 5 | 3,477.5 | $33{ }^{\circ}$ | 11,196 | 3,957 | 21.911 | 1,301 | 14.566 | 1,957 | 21.911 |
| 6 | 5,712.5 | . 369 | 11,196 | 2,318 | 25.952 | 1,421 | 15.910 | 2,214 | 24.788 |
| 7 | 5,442.5 | 369 | 10,827 | 3,915 | 42.388 | 1,421 | 15.3985 | 2,214 | 23.971 |
| 8 | 5,625.0 | 196 | 10,827 | 1,395 | 15.104 | 755 | 8.174 | 1,176 | 12.732 |
| 9 | 2,817,5 | 326 | 10,827 | 2,205 | 23.874 | 1,255 | 13.588 | 1,956 | 21.178 |
| 10 | 2,527.5 | 267 | 11,196 | 2,182 | 24.430 | 935 | 10.468 | 1,442 | 16.145 |
| 1) | 2,490,0 | 322 | 11,196 | 2,048 | 22.929 | 1,127 | 12.618 | 1,739 | 19.470 |
| 12. | 2,527.5 | 470 | 11.167 | -1,935 | 21.608 | 1,645 | 18.370 | 1,935 | 21.608 |
| 13 | 1,780.0 | 315 | 11,167 | 1,485 | 16.583 | 1,102 | 12.308 | 1,485 | 16.583 |
| 14 | 2,615.0 | 430 | 11,167 | 2,250 | 25.126 | 1,505 | 16.806 | 2,250 | 25.126 |
| 15 | 2,895.0 | 362 | 11,008 | 2,520 | 27.740 | 1,340 | 14.751 | 2,172 | 23.909 |
| 16 | 5,405.0 | 129 | 1],008 | 2,565 | 28.236 | 497 | 5.471 | 774 | 8.520 |
| 17 | 5,505.0' | 329 | 11,008 | 3,240 | 35.666 | 1,267 | 13.947 | 1,974 | 21.730 |
| 18 | 4,152.5 | 374 | 11,008 | 3,712 | 40.862 | 1,440 | 15.851 | 2,244 | 24.702 |
| 19 | 3,937.5 | 456 | 11,008 | 3,420 | 37.647 | 1,756 | 19.330 | 2,736 | 30.118 |
| 20 | 2,027.5 | 268 | 11,008 | 1,800 | 19.814 | 1,032 | 11.360 | 1,608 | 17.701 |
| 21 | 2,962.5 | 245 | 11,008 | 2,272 | 25.010 | 943 | 10.381 | 1,470 | 16.18! |
| 22 | 4,590.0 | 652 | 11,008 | 2,453 | 27.003 | 2,453 | 27.003 | 2,453 | 27.003 |
| 23 | 9,827.5 | 146 | 11,008 | 1,980 | 21.796 | 562 | 6.186 | 876 | 9.643 |
| 24 | 6,407.5 | 78 | - | - | - | - | - | - | - |
| 25. | 16,680.0 | 54 | - | . - . | - | - | - | - | - |
| 26 | 7,227.5 | 274 | 11,008 | 2,520 | 27.740 | 1,055 | 11:613 | 1,644 | 18.097 |
| 27. | 14,902,5 | 56 | - | - | . - | , - | - | - | - |
| - I | 141,170.0 | 7,986 | - | 57,487 |  | 29,212 |  | 42,987? |  |

[^2]
### 8.4 Conveyance and field irrigation losses

In developing the linear programming model, we had to take into account the water that is inevitably lost from an irrigation canal system. Obviously, the farther the water is conveyed, the greater will be the losses. The quantity of surface water eventually arriving at a group inlet (which is assumed to supply farm groups of 50 ha ) will thus be less than the quantity released from the diversion dam. Downstream of the group inlet, more water is lost from the sublaterals and field ditches, and on the fields.

Although a certain amount of water will be lost through percolation, what we were most interested in were the losses that percolate to the water table and add to the recharge of the groundwater basin. So, disregarding evaporation losses for the time being, what we had to find was:

1. what fraction of a unit volume of surface water released from the diversion weir is eventually available for irrigation on the farm? Knowing this, we could calculate how much water had to be released fron the weir to meet the water demand of a polygon, and how much was lost through percolation from the canal system.
2. what fraction percolates to the water table of a polygon downstream of the group inlet? This fraction defines the maximum quantity of groundwater that can be withdrawn from that polygon.

The sum of the percolation losses increases the maximum safe yield of the basin and thus indicates the total quantity of groundwater that can safely be withdrawn. Hence, our concern was to find the numerical values of the coefficients $b, c$, and $e$, as defined in Chap.6, Section 3.

### 8.4.1 Conveyance losses in main canals and laterals

It will be clear that in calculating the percolation losses in the canal system, we had to make certain assumptions. One was that the main canals and laterals would be lined, and another was that the percentage of water lost by percolation would be:

| lined main canals | $0.5 \%$ per km |
| :--- | :--- |
| lined laterais | $1.0 \%$ per km |

We then had to determine these losses on a polygonal basis. A complicating factor was that certain polygons would be supplied by two main canals, so that we first had to calculate the flow through each canal.

Because of space limitations, we shall refrain from reproducing the lengthy calculations that were made. The following example, however, may serve to illustrate how we determined the losses and the net quantity of surface water that eventually arrives at a farm group inlet.

Let us choose polygon 18, whose water demand will be met by water from Main Canal V, which crosses the polygons i, 4, 11, 14, and 18, and from Main Canal V-2, which crosses the polygons $1,5,10,15$, and 18 (see Fig.29).

As can be seen from the figure, there is a partition point in polygon $1,4.1 \mathrm{~km}$ from the diversion weir. At that point, Main Canal V-2 branches off from Main Canal V. Of the water available there, 15 per cent is conveyed through Canal $V$ and 85 per cent through Canal $V-2$.

The water losses from these canals were calculated as follows. Suppose a flow Qe enters a 1 km stretch of canal. Because of seepage losses in that stretch of canal, the flow gradually diminishes, and at the end of the stretch a flow $Q_{1}$ is left. The flow $Q_{l}$ leaving the 1 km stretch of canal now becomes the flow $Q_{e}$ entering the next canal stretch of 1 km , and so forth.

In this manner we calculated the accumalated water losses as a percentage of the initial flow released from the diversion weir, assuming 0.5 per cent percolation losses per km in main canals and 1.0 per cent per kn in laterals. The results are given in Tables 39 and 40.

To calculate the fraction of a unit volume of flow released from the diversion weir and actually reaching polygon 18 , the lengths of the canals crossing the above polygons were measured on Fig. 29 .

With the type and the lengths of the canals known, we could select from Tables 39 and 40 the corresponding percentages of the initial flow and then calculate the fraction of the initial unit volume of surface water leaving a polygon. This fraction became the flow entering the next downstream polygon, and the

TABLE 39. Accumulated percolation losses in main canals (losing 0.5 per km )

| Length of canal segment (ks) | Percentage of the initial flow | Water losses in the segment | Accumulated water losses |
| :---: | :---: | :---: | :---: |
|  | leaving the segment | in 2 of the | initial flow |
| 0 | 100.00 | 0.500 | 0.500 |
| 1 | 99.50 | 0.497 | $0.997^{\circ}$ |
| 2 | 99.00 | 0.495 | 1.492 |
| 3 | 98.51 | 0.493 | 1.985 |
| 4 | 98.02 | 0.490 | 2.475 |
| 5 | 97.53 | 0.488 | 2.963 |
| 6 | 97.04 | 0.485 | 3.448 |
| 7 | 96.55 | 0.483 | 3.931 |
| 8 | 96.07 | 0.480 | . 4.411 |
| 9 | 95.59 | 0.478 | 4.889 |
| 10 | 95.11 | 0.476 | 5.365 |
| 11 | 94.63 | 0.473 | - 5.838 |
| 12 | 94.16 | . |  |

a.s.o.

TABLE 40. Accumulated percolation losses in laterals (losing $1.0 \%$ per km )

| Length of canal segment (km) | Percentage of the inicial flow | Water losses in the segment | Accuanlated water losses |
| :---: | :---: | :---: | :---: |
|  | leaving the segment | in 7 of the | aitial-flow |
| 0 | 100.00 | 1.00 | 1.00 |
| I | 99.00 | 0.99 | 1.99 |
| 2 | 98.01 | 0.98 | 2.97 |
| 3 | 97.03 | 0.97 | 3.94 |
| 4 | 96.06 | 0.96 | 4.90 |
| 5 | 95.10 | 0.95 | 5.85 |
| 6 | 94.15 | 0.94 | 6.79 |
| 7 | 93.21 | $0: 93$ | 7.72 |
| 8 | 92.28 | 0.92 | 8.64 |
| 9 | 91.36 | 0.91 | 9.55 |
| 10 | 90.45 |  |  |

[^3]table 41. example of calculating per polygon the net deep percolation losses prom main canals and LATERALS, IF ONE UNIT VOLUME OF SURFACE WATER IS RELEASED FROH THE DIVERSION WEIR TO LATERALS, IF ONE
SUPRLY POLYGON 18

| Columm | 2 | 2 | 3 | 4 | $s$ | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Polygen crossed by surface water | Fraction of a unit volume of water enterith polygon | Length of canal in polygon <br> (km) | Percolation loss per ks of capal <br> (Z) | Percencage of initial flow Cor given canal length (bee Tables 22 \& 23) | Fraction of initial unit volume of water leaving polygon $(\operatorname{Col} .1 \times 4)$ | Fraction of initisl unit volume of vater lost in crossed polygon (Col.1-5) | Fraction of initial unit volume of water available at farm group inlet | Net deep percolation from ebnals in polygon $(\operatorname{Col} .6 \times 0.80)$ |


calculations were repeated. The fraction of water that percolated in a crossed polygon was found as the difference between the fractions of the initial volume of water entering and leaving the polygon in question.

The results of the calculations for polygon 18 are shown in Table 41 .

It can be seen from that table that, if we release a unit volume of surface. water from the diversion weir at the apex of the alluvial fan to supply polygon 18 , there will eventually be 0.8341 of that unit volume available at the group inlet:

Although percolation losses will certainly add to the groundwater recharge, it is not realistic to assume that all water lost from the canals and laterals will percolate to the water table. Part of it will be retained by the soil, another part will evaporate. As an average we therefore assumed that 80 per cent of the gross percolation losses would actually reach the water table.

Hence, to obtain the net deep percolation from the canals and laterals, we multiplied the values in Column 6 of Table 41 by 0.80 . The results are presented in Column 8. The sum of the net deep percolation losses (e) in all the various polygons is 0.1326 . This value must be understood to mean the contribution to the recharge of the groundwater basin as a whole, if a unit volume of surface water is released from the diversion weir to supply polygon 18.

In the same way as described above, we calculated the fractional losses and the fractions available at each farm group inlet for all the other polygons.

### 8.4.2 Percolation losses downstream of farm group inlet

Once the surface water has reached the farm group inlet, it will be conveyed to the fields through sublaterals and farm ditches. Here again, we made assumptions of the percolation losses in these distributaries and on the fields (see Fig. 33).

Let the quantity of surface water available at the farm group inlet be equal to 100 per cent. We assumed that 10 per cent of this would be lost in the sublaterals. Of this 10 per cent, $2 \frac{1}{2}$ per cent, say, will percolate to the water table and the remaining $7 \frac{1}{2}$ per cent will be lost in another way (evaporation).


Fig.33. Scheme of assumed water losses, Group inlet supplies farm groups of 50 ha.

The water available at the farm is therefore 90 per cent of that at the farm group inlet. Of this 90 per cent, 30 per cent will be lost in different ways and the remaining 60 per cent will be used consumptively by the crops. We assumed that the 30 per cent water losses on the farm are made up of $2 \frac{1}{2}$ per cent evaporation loss, 20 per cent runoff, and $7 \frac{1}{2}$ per cent deep percolation loss.

Downstream of the farm group inlet we thus have a total net deep percolation of 10 pet cent of the water available at the group inlet ( $2 \frac{1}{2}$ per cent in the sublaterals $+7 \frac{1}{2}$ per cent on the farm).

In our example of polygon 18, we calculated that, of a unit volume of water released from the diversion weir, 0.8341 will be available at that polygon's group inlet, and 0.1326 will percolate to the water table in the polygons crossed upstream of the inlet. Downstream of the inlet, 10 per cent of 0.8341 will percolate to the water table, or 0.0834 . Hence, a unit volume of surface water released from the diversion weir to supply polygon 18 will contribute
$0.1326+0.0834=0.2160$ to the groundwater recharge of the basin. The maximum "safe yield" of the basin (MXSFY) will thus be enlarged by $b * 0.2160$ (see the second constraint in Chap.6, Sect.3).

This value; with a minus sign, was entered in the 1 inear programing matrix under the column SRF of polygon 18 and the row MXSFY (see Table 45).

The quantity of surface water available for irrigation on the farm was assumed to be 90 per, cent of the quantity available at the group inlet. For polygon 18 this is 90 per cent of 0.8341 , or $c=0.7506$ (see the fourth constraint in Chap. 6, Sect. 3).

In the same way as described above the numerical values of $b$ and $c$ were calculated for all the other polygons, except polygon 23 , whose water demand will be met by well water. Tables 42 and 43 sumarize the results of these calculations.

As defined earlier, $e_{i j}$ is the portion of the surface water that percolates to the water table in $i$ polygons, if a unit valume of water is released from the diversion weir to supply polygon $j$. If $i \neq j$, then $e_{i j}$ represents the deep percolation in the main canals (and laterals) in the $i$ polygons. If $i=j$ then $e_{i j}$ represents the cotal deep percolation from the main canal (if it occurs in the $j^{\text {th }}$ polygon), laterals, sublaterals, and the deep percolation on the farms in the $j^{\text {th }}$ polygon.

As an example, let us take polygon 4 . From 'Table 42 it can be seen that the frac-. tional deep percolation from the main canals as fat downstream as the last offtake in polygon 4 and that from the laterals, totals 0.0140. To this must be added the deep percolation from the sublaterals and that occurring on the farms, i.e. 10 per cent of the water available at the farm group inlet or 0.0951 (see Table $43)$. Hence $e_{44}=0.0140+0.0951=0.1091$.

TABLE 42. Values of the coefficient efor the different polygons

| From weir to po1ygon | $\begin{aligned} & \text { Crossed } \\ & \text { polygons } \\ & \text { (sie } \\ & \text { Fig.29) } \end{aligned}$ |  |  |  |  | Fory boorumber |  |  |  |  |  |  |  | total. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | \% | 10 | 11 | 12 |  |
| 1 | 1 | 0.0192 |  |  |  |  |  |  |  |  |  |  |  | 0.0192 |
| 2 | 1,2 | 0.0276 | 0.0304 |  |  |  |  |  |  |  |  |  |  | 0.0580 |
| 3 | 1,2,3,4 | 0.0276 | 0.0533 | -0.0187 | 0.0059 |  |  |  |  |  |  |  |  | 0.1055 |
| 4 | 1,2,4 | . 0.0241 | 0.0008 |  | 0.0140 |  |  |  |  |  |  |  |  | 0.0389 |
| 5 | 1,4,5 | 0.0287 |  |  | 0.0052 | 0.0128 | . |  |  | - |  |  |  | 0.0467 |
| 6 | 1,6 | 0.0309 |  |  |  |  | 0.0410 |  |  |  |  |  |  | 0.0719 |
| ? | 1,5,6,7 | 0.0204 |  |  |  | 0.0013 | 0.0373 | 0.0198 |  |  |  |  |  | 0.0785 |
| 8 | 3,6,7,8 | 0.0198 | . |  |  |  | 0.0438 | 0.0393 | 0.0114 |  |  |  |  | 0.1143 |
| 9 | $\begin{aligned} & 1,5,6,7 \\ & 8,9,10 \end{aligned}$ | 0.0292 |  |  |  | 0.0214 | 0.0072 | 0.0075 | 0.0015 | 0.0085 | 0.0160 |  |  | 0.0972 |
| 10 | $\begin{aligned} & 1,4,5, \\ & 10,11 \end{aligned}$ | 0.0278 |  |  | 0.0106 | 0.0143 |  |  |  |  | 0.0126 | 0.0136 |  | 0.0789 |
| 11 | 1,4,31 | 0.0237 |  |  | 0.0230 |  |  |  |  |  | . | 0.0405 |  | 0.0872 |
| 12 | 1,4,11,12 | 0.0237 |  |  | 0.0230 |  |  |  |  |  |  | 0.0260 | 0.0286 | 0.1013 |

a.s.0.

TABLE 43. Values of the coefficients $b$ and $c$ for the different polygons


The values of the coefficient $e_{i j}$ for $i=j$ were calculated and the results are given in Table 44.

| TABLE 44. | Values of the coefficients <br> (for $i=j)$ | $e_{i j}$ <br> Polygon No. <br> 1$e_{i j}$ | Polygon No. |
| :---: | :---: | :---: | :---: |$e_{i j}$

### 8.5 Linear programming matrix

Once we knew the values of the coefficients, the costs of the activities (agricultural production, well water supply, and surface water supply), and the values of the constraints, we could then enter them in the linear programming matrix, as was shown in Table 29. From this, we were able to compile the master matrix of the model, part of which is shown in Table 45.

TABLE 45. Part of the linear programming matrix

| Activities |  |  |  | $\begin{array}{r} 01 \\ \mathrm{PRD} \end{array}$ | $\begin{gathered} 01 \\ \text { WEL } \end{gathered}$ | $\begin{gathered} 01 \\ \text { SRF } \end{gathered}$ | $\begin{array}{r} 02 \\ \text { PRD } \end{array}$ | $\begin{array}{r} 02 \\ \text { WEL } \end{array}$ | $\begin{array}{r} 02 \\ \mathbf{S R F} \end{array}$ | $\begin{array}{r} 03 \\ \text { PRD } \end{array}$ | $\begin{array}{r} 03 \\ \text { WEL } \end{array}$ | $\begin{array}{r} 03 \\ \text { SRF } \end{array}$ | $\begin{array}{r} 04 \\ \text { PRD } \end{array}$ | $\begin{array}{r} 04 \\ W E L \end{array}$ | $\begin{gathered} 04 \\ \text { SRF } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Costs (Rials) |  |  |  | -2. 204 | 1.0073 | 0.1833 | -1.867 | 0.6901 | 0.3058 | -1.867 | 0.4593 | 0.2814 | -2.670 | 0.6110 | 0.2567 |
| Constraints (million m    <br> 3)    <br> MXRIV 150.0 220.0 340.0 <br> MXSFY 60.0 60.0 60.0 <br>  RHS 1 RHS 2 RHS 3 |  |  |  |  | $+0.9$ | $\begin{aligned} & +1.0 \\ & -0.1168 \end{aligned}$ |  | $+0.9$ | $\begin{aligned} & +1.0 \\ & -0.1508 \end{aligned}$ |  | $+0.9$ | $\begin{aligned} & +1.0 \\ & -0.1923 \end{aligned}$ |  |  | $\begin{aligned} & +1.0 \\ & -0.1340 \end{aligned}$ |
| 01 MXD <br> DEM <br> MXSFY | $\begin{aligned} & 4.793 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{aligned} & 4.793 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{aligned} & 4.793 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{aligned} & +1.0 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -1.0 \\ & +0.9 \end{aligned}$ | $\begin{aligned} & -0.8784 \\ & -0.1168 \end{aligned}$ |  |  | -0.0276 |  |  | -0.0276 | . |  | -0.0241 |
| 02 MXD <br> DEM <br> MXSFY | $\begin{gathered} 32.646 \\ 0.0 \\ 60.0 \end{gathered}$ | $\begin{aligned} & 18.855 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{gathered} 29.378 \\ 0.0 \\ 60.0 \end{gathered}$ |  |  |  | $\begin{aligned} & +1.0 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -1.0 \\ & +0.9 \end{aligned}$ | $\begin{aligned} & -0.8348 \\ & -0.1232 \end{aligned}$ |  | $\cdot$ | -0.0533 |  |  | -0.0008 |
| 03 MXD <br> DEM <br> MXSFY | $\begin{aligned} & 47.920 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{aligned} & 19.316 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{aligned} & 30.88! \\ & 0.0 \\ & 60.0 \end{aligned}$ |  |  |  |  |  | . | $\begin{aligned} & +1.0 \\ & +1.0 \end{aligned}$ | $\begin{aligned} & -1.0 \\ & +0.9 \end{aligned}$ | $\begin{aligned} & -0.7814 \\ & -0.1055 \end{aligned}$ |  |  |  |
| 04 MXD <br> DEM <br> MXSFY | 13.104 <br> 0.0 <br> 60.0 | $\begin{aligned} & 3.724 \\ & 0.0 \\ & 60.0 \end{aligned}$ | $\begin{aligned} & 5.803 \\ & 0.0 \\ & 60.0 \end{aligned}$ |  |  |  |  |  |  |  |  | $-0.0059$ | $\begin{aligned} & +1.0 \\ & +1.0 \end{aligned}$ | $\begin{array}{r} -1.0 \\ +0.9 \end{array}$ | $\begin{array}{r} -0.8562 \\ \cdot-0.1091 \end{array}$ |

05 a.s.o.

## 9. Procedure in using the models

Once we had the models available, the procedure we followed was first to use the linear programing model to calculate the optimal solutions for different schemes of water allocation in the Plain. Since the model was developed to generate, polygon-wise, the net deep percolation values accompanying each solution, these values were fed into the groundwater model, which then simulated the percolations and generated the water table elevations for each polygon for, say, a period of 10 or 40 years, as was required.

The next step was to examine the water table changes. This was done by drawing, for each polygon,a graph of the water table elevations against time (hydrographs) and a map of the water table changes that would be found at the end of the period considered. We could then see in which polygons unacceptable changes would occur.

The process of bringing the water table inder control in those polygons was one. of trial and error. If, due to excessive percolation, the water table in a particular polygon rose to, or very near to, the surface, the surface water supply to that polygon was reduced by an estimated portion and the remaining water requirement was' supplied by wells. Similarly; if excessive pumping from wells caused the water table in a polygon to drop too fast and too deep, the pumping rate was reduced by an estimated portion and the remaining water requirement was met by surface water.

Such adjustments were then imposed on the linear programming model and a new computer run was made, resulting in a new, sub-optimal water supply solution.

The solution was then tested on the groundwater model for the effect it would have on the water table in the individual polygons and in the basin as a whole. If the adjustments were found to be satisfactory, the water supply solution was considered feasible. If, on the other hand, some of the water table changes wete still too great, further adjustments were made and computer runs were repeated as many times as were necessary to obtain a solution that was feasible. In general only a few computer runs were needed.

### 9.1 Determining the maximum river discharge .

The basic problem was to find the maximum hectarage of land that could be irrigated and kept under irrigation even in water-deficient years, given the stochastic river flow and a maximum annual rate of groundwater abstraction.

Throughout our study we assumed, as an acceptable risk, that only once in 20 years would the river discharge be less than that required to meet demands. (We assumed that in such a dry year the farmers would be subsidized.) This risk being accepted, the lower limit of the river discharge was defined as $150 \times 10^{6} \mathrm{~m}^{3}$ a year (see Chapt.2, Section 6.1). It can be seen from Figs. 6 and 7 , that in 19 years out of 20 , that river discharge will be exceeded,

A scheme based on this minimum river discharge would mean that the groundwater basin could be operated on a "safe yield" basis, i.e. no overdraft would be necessary in water deficient years because such years would not occur; nor would there be any need to recharge the groundwater basin artificially.

In many of the 19 years, however, the river discharge would be considerably bigher than $150 \times 10^{6} \mathrm{~m}^{3}$ a year, and all the excess flow would simply run away unused. Such waste is unacceptable.

Now the problem was to find the maximum river discharge which, in combination with a maximum annual rate of groundwater abstraction, would allow the maximum hectarage of land to be itrigated and kept under irrigation even in water-deficient years.

As a first step in trying to solve this problem we applied what is known as a parametric linear programming approach (HALL and DRACUP, 1970). This approach allows the effect on solutions to a linear programming problem to be studied, when certain known constraints are allowed to vary. For this purpose we selected the water demand constraint RHS I (allocating water to the best soils) and the resource constraint maximum "safe yield" (MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, i.e. the groundwater basin was not allowed to be mined) and combined them with the maximum river discharge constraint (MXRIV); this we allowed to vary, while keeping the other two constraints constant. For each new computer run we increased the value of NXRIV by a constant quantity: $30 \times 10^{6} \mathrm{~m}^{3}$, except for the first interval for which MXRIV was increased by $70 \times 10^{6} \mathrm{~m}^{3}$.

Table 46 shows a schematic of the parametric linear programming runs made.

TABLE 46. Parametric linear-programming runs

| MXRIV <br> $\left(\times 10^{6} \mathrm{~m}^{3}\right)$ | MXSFY <br> $\left(\times 10^{6} \mathrm{~m}^{3}\right)$ | Water demand | Irrigated area |
| :---: | :---: | :---: | :---: |
| 150 | 60 |  | (ha) |
| 220 | 60 | RHS 1 | 19,409 |
| 250 | 60 | RHS I | 25,857 |
| 280 | 60 | RHS 1 | 28,606 |
| 350 | 60 | RHS 1 | 31,379 |
| 340 | 60 | RHS 1 | 34,150 |
|  |  |  | 36,883 |

${ }^{1}$ see also Table 38

For each of the combinations shown in Table 46 the optimal water supply solution was obtained and the value of the objective function calculated. The study had no other meaning than to appreciate the economic consequences of having more and more water available to the Plain,e.g. by importing foreign water, whose quantity is not yet precisely known. The expansion of the water allocation to the Plain could be appreciated from the study, i.e. the hectarage of land that could be irrigated for each given river discharge and the assumed constant groundwater abstraction. These hectarages were calculated and are given in Table 46.

Returning now to our problem of finding the maximum river discharge, we have shown that a scheme based on a river flow of only $150 \times 10^{6} \mathrm{~m}^{3}$ a year is unacceptable. The next value we studied, was $220 \times 10^{6} \mathrm{~m}^{3}$ a year. As can be seen from Table 46 , this discharge nould allow 25,857 ha to be irrigated, a hectarage which corresponds roughly with the average area that was under irrigation at the time of our study.

From the statistical analysis discussed in Chap.2, Section 6.1, we knew that in 4 years out of 20 the river discharge of $220 \times .10^{6} \mathrm{~m}^{3}$ would not be exceeded. This meant that, accepting a risk of 1 year of water deficiency, the groundwater basin would have to be used for 3 years under overdraft conditions to overcome. the water shortage, if the same hectarage of land as in the other 16 years was to be maintained.

For a water-deficient year, we took as an average river discharge $150 \times 10^{6} \mathrm{~m}^{3}$, a rather conservative figure. We imposed this discharge on the linear programning 140
model, released the maximum "safe yield" constraint, and obtained the optimal solution of water allocation.

In the next step, we considered a 40 -year period and applied cycling of river discharges: in the first 16 years an annual river discharge of $220 \times 10^{6} \mathrm{~m}^{3}$ was. taken, with the groundwater basin operating on a "safe yield" basis. This was followed by 4 years with an average annual river discharge of $150 \times 10^{6} \mathrm{~m}^{3}$, with the maximum "safe yield" constraint released to maintain the level of agricultural production of the previous 16 -year period. This sequence was repeated to make up the 40 years.

The groundwater madel was then used to simulate this river discharge cycle and its corresponding cycle of net deep percolation. For the entire 40 -year period, we examined the annual changes in the water table in each polygon and found a long-term decline.

In the final step we made an attempt to cure this defect by imposing an artificial recharge of $8 \times 10^{6} \mathrm{~m}^{3}$ a year in each of the three northern polygons. This had a marked stabilizing effect on the water table and the scheme was found to be feasible.

The next logical step in the procedure was to examine whether a maximum river discharge higher than the previous one would also provide a feasible scheme. We could have worked through the range of river discharges indicated in Table 46 until a schene was found that was not feasible. We did not do so, however, but repeated the procedure for only one maximum river discharge: $340 \times 10^{6} \mathrm{~m}^{3}$ a year. It was found that a scheme based on this rather high river discharge was impossible because of the substantial long-term decline in the water table. The conclusion was that the maximum river discharge to be accepted must be somewhere. between $220 \times 10^{6} \mathrm{~m}^{3}$ and $340 \times 10^{6} \mathrm{~m}^{3}$ a year. Probably the maximum will be slightly higher than $220 \times 10^{6} \mathrm{~m}^{3}$, but no further attempts were made to determine it precisely. The reason was that we had to work with incomplete data and,besides; the emphasis in this study was more on methodology than on practical results.

### 9.2 Determining the maximum groundwater abstraction

In Chapter 6, Section 3, we defined the "safe yield" concept used in this study as the difference between the subsurface flow into the basin and the sub-
surface flow out of the basin. It was found to be of the order of $60 \times 10^{6} \mathrm{~m}^{3}$ a year, a quantity that can be withdrawn from the underground annually without exhausting the resource. In addition, all the surface water reaching the water table from canal and field percolation can be pumped (see Constraint No.2, Chap. 6, Section 3). In water-deficient years - and these will be more frequent as we choose higher river discharges - the above permissible rate of groundwater abstraction will not be sufficient to overcome the water shortage. In such years the groundwater must be mined. This was done in the model by releasing the maximuri "safe yield" constraint of $60 \times 10^{6} \mathrm{~m}^{3}$ a year. In a water-deficient year as much groundwater would be pumped as was needed.

Operating the groundwater basin under alternating conditions of "safe yield" and overdraft would cause a long-cerm drop in the water table. This means that the groundwater basin must be recharged artificially by part of the river water that is in excess of the discharge selected.

The questions we had to answer were: how many years can the groundwater be mined and at what rate, and what quantity of excess river water can be used for the artificial recharge of the basin and at what places can this best be done. Once we had found the answers to these questions, which were found by trial and error, we could determine the maximum river discharge that should be accepted for the project and, knowing this, we could determine the maximum hectarage of land that could be irrigated and kept under irrigation in the future.

It should be noted that the problem of artificial recharge was studied from a bydrological viewpoint only, i.e. the effect the recharge would have on the water table. Because of 1ack of time and funds, we could not perform any actual recharge tests nor make a cost analysis of this activity. Consequently we were unable to optimise it economically.

The most suitable site for artificial recharge of the basin was found to be the gravel fields at the head of the alluvial fan. The simulations were therefore performed for this area only.

### 9.3 Selecting a land-use policy

An important decision variable that we built into the linear programming model was the use of the available land resources. As was explained in Chap.6, this 142
variable was denoted as RHS and three different policies were defined: allocating water to the best land only regardless of the number of farmers living in the Plain (RHS I), allocating water to all the farmers in the Plain and each farmer possessing 3.85 ha of land (RHS 2), and allocating water to all the farmers in the Plain and each farmer possessing 6 ha of land (RHS 3).

A decision in favour of one of these alternatives is a socio-economic problem. From a purely economic viewpoint, the first alternative would yield the highest. value of the objective function (total net benefit). At present quite a lot of good land lies temporarily fallow, so that RHS $i$ would imply the resettlement of a number of farmers from areas with marginal soils to those areas with the best soils.

The second altenative is, economically speaking, less attractive because we can expect a lower value of the objective function. The resettlement problem, however, would be less severe.

Finally, the third alternative would pose a very severe sociological problem, as many farmers would not be supplied with water and consequently would have to give up their profession. Although such a development is not unthinkable, we considered this alternative merely as a possible phase in the future development, of the glain. One might, for instance, think of making more water available to the Plain by importing foreign water. This would allow much more land to be irrigated than at present and the size of the farms could be enlarged.

Merely for the sake of showing decision makers the consequences of these policies, we studied a scheme in which a maximum river discharge of $220 \times 10^{6} \mathrm{~m}^{3}$ a year was combined with the water demand constraint defined as RHS 2 . The same stepwise procedure was followed as before and a feasible solution of water supply was obtained.

The economic consequences of resettling a number of farmers was not further studied, but for each feasible solution we calculated the value of the objective function, the hectarage of land that could be irrigated and that which could not, for each polygon individually and for the entire Plain. Since the number of farmers in each polygon was known, those that would receive water and those that would not were calculated too.

### 9.4 Schematic of the computer studies

'Table 47 summarizes the computer studies made.

TABLE 47. Summary of the various schemes studied by the models

| Scheme <br> (No.) | $\begin{gathered} \text { MXRIV } \\ \left(\times 10^{6} \mathrm{~m}^{3}\right) \end{gathered}$ | $\begin{gathered} \text { MXSFY } \\ \left(\times 10^{6} \mathrm{~m}^{3}\right) \end{gathered}$ | $\begin{aligned} & \text { MXD } \\ & \text { (RHS) } \end{aligned}$ | Tested by the groundwater model |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 220 | 60 | 1 | yes |
| 2 | 150 | $>60$ | 1 | yes |
| 3 | 220/150 | $60 />60$ | ! | yes |
| 4 | 220/150 + artificial recharge | $60 />60$ | 1 | yes |
| 5 | 340 | 60 | 1 | yes |
| 6 | 220 | > 60 | 1 | no |
| 7 | 150 | $>60$ | 1 | no |
| 8 | 340/220/150 | $60 />60 />60$ | 1 | yes |
| 9 | 220 | 60 | 2 | yes |
| 10 | '150 | $>60$ | 2 | yes |
| 11 | 220/150 | 60/>60 | 2 | yes |
| 12 | $220 / 150+$ artificial recharge | 60/>60 | 2 | yes |
| 13 | 220 | 60 | 1 | no |
| 14 | 250 | 60 | 1 | no |
| 15 | 280 | 60 | 1 | no |
| 16 | 310 | 60 | 1 | no |
| 17 | 340 | 60 | 1 | no |

Note: Schemes Nrs. 13 to 17 are parometric linear progranming muss

## 10. Results obtained from the modelling studies

### 10.1 Water supply scheme No. 1

### 10.1.1 Optimal solution

The first water supply scheme we investigated was that of supplying water only to the best land, i.e. those of Class I and Class II (Fig.5). This policy is indicated in the matrix as RHS 1. The relevant maximum water demand values for each polygon are given in Table 38.

The scheme is also based on the assumption that $220 \times 10^{6} \mathrm{~m}^{3}$ of surface water would be available annually. As was shown earlier this would be true in 16 years out of 20 .

It was further assumed that the groundwater basin would be operated on a "safe yield" basis,i.e. the total groundwater abstraction would not exceed $60 \times 10^{6} \mathrm{~m}^{3}$ a year, except for that portion of the surface water that would percolate to the water table.

The optimal water supply solution produced by the linear programming model for this scheme is shown in Table 48 and Fig. 34. The solution indicates that the land that would be supplied with irrigation water represents a rather concentrated block, located in the northern and middle parts of the Plain, The northern and middle nine polygons, which form a cluster, would be supplied with surface water, and polygons 3, 12 (partly), 13,21 , and 22 with well water. With this solution all available surface water and groundwater would be used and no water would be available to the peripheral polygons, in many of which farmers are living. These farmers would have to be moved to the northern and middle part of the Plain, to those areas of Class I and Class II land, which are now only partly cultivated.

As can be seen from Fig. 34, polygon 20 will not receive any water at all. The reason for this is that all available surface water has already been used in the upstream polygons, and that the $\mathrm{m}^{3}$ price of well water in this polygon is slightly higher than that in the adjacent polygons 19, 21 , and 22 (see Table 37). Since the net return per $\mathrm{m}^{3}$ of water in polygon 20 is the same as in the adjacent

TABLE 48. Optimal water supply solution for Scheme No. 1. MXRIV $=220 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and MXD $=$ RHS 1


Net benefit R1s 615,076,900


Fig.34. Optimal water supply solution of Scheme No.1.
ones (R1s 1.867 per $\mathrm{m}^{3}$, see Table 22 and Fig.27), the computer did not include polygon 20 in the solution.

It is obvious that if this scheme were to be considered for implementation, such an irregularity would have to be smoothed out, because other important factors, such as territorial homogeneity (the area to be developed should preferably form a siagle block), existing farms and villages, higher infra-structural investments, etc., may outweigh the higher well water costs in polygon 20.

Table 48 also shows that of the cotal water demand of 633.070 million $\mathrm{m}^{3}$ only $45.2 \%$ would be met ( 286,471 million $\mathrm{m}^{3}$ ), for the remaining $54.8 \%$ no more water is available: there is an unsupplied demand of 346.600 million $\mathrm{m}^{3}$.

Of the total water quantity supplied, $37.7 \%$ is well water and $62.3 \%$ surface water.

The last column of Table 48 shows the gross quantitites of surface water released from the diversion weir. If we multiply these polygonal quantities with the corresponding coefficients of Column 5 from Table 43 , we obtain the quantity that each polygon supplied with surface water contributes to the "safe yield" of the basin. The sum of these percolations is 37.112 million $\mathrm{m}^{3}$. To this we add 60.000 milition $\mathrm{m}^{3}$ and obtain 97.112 million $\mathrm{m}^{3}$, which represents the actual "safe yield" of the basin. The cotal groundwater pumpage is 107.902 million $\mathrm{m}^{3}$ (Table 48). As stated earlier, we assumed a return flow of $10 \%$ for the well water, so that the net pumpage is therefore $90 \%$ of 107.112 or 97.112 million $\mathrm{m}^{3}$, as calculated above.
10.1.2 Testing the optimal solution for its technical feasibility

From an economic viewpoint the water supply solution obtained is optimal, but it was still uncertain whether, if implemented, it would not create unacceptable water table changes in parts of the Plain. The solution was therefore tested by the groundwater model for the impact it would have on the water table.

This was done by merely replacing the set of 27 initial net deep percolation values (AQ-values) by a new set, representing the polygonal net deep percolation that would occur if the optimal water supply scheme were implemented. The groundwater model simulated these new $A Q-v a l u e s$ and predicted the water table elevations for each polygon and for any chosen period, say 10 years.

As can be seen from the matrix (Table 45), each polygon contained a row that represented the maximum "safe yield" of the polygon, denoted as MXSFY. Initially 148


Fig. 35. Portion of the linear programming output of Scheme No. 1.
we had, rather arbitrarily, assigned the value of $60 \times 10^{6} \mathrm{~m}^{3}$ to the maximum "safe yield" of all polygons; in ocher words, the maximum "safe yield" of a polygon was equal to that of the basin as a whole (see Table 45 , second row under Constraints). This meant that we could theoretically pump all available groundwater from one single polygon. This would never occur, however, because the maximum water demand of a polygon for RHS 1 is considerably less than $60 \times 10^{6} \mathrm{~m}^{3}$ a year (Table 38). Any percolation of surface water in a particular polygon (canal and/or field percolation) contributes to its maximum "safe yield" and is therefore added to its initially assigned value of $60 \times 10^{6} \mathrm{~m}^{3}$. On the other hand, any groundwater pumpage in a particular polygon reduces the maximum "safe yield" of that polygon and is therefore subtracted from the initial $60 \times 10^{6} \mathrm{~m}^{3}$.

Figure 35 shows a portion of the linear programming output from Water Supply Scheme No.1. The net deep percolation in each polygon, could easily be derived from this output, being the difference between the printed value and the initial polygonal maximum "safe yield" of $60 \times 10^{6} \mathrm{~m}^{3}$.
Constraint No.77, for example, has increased from $60 \times 10^{6} \mathrm{~m}^{3}$ to $66.358 \times 10^{6} \mathrm{~m}^{3}$, or by $6.358 \times 10^{6} \mathrm{~m}^{3}$. This is the quantity that would percolate to the water table in polygon 1 if surface water is supplied to this polygon (canal and field percolation) and to downstream polygons (canal percolation from surface water transported through polygon 1 to supply downstream polygons).

Constraint No. 85 refers to polygon 3 and it can be seen that its value is reduced from $60 \times 10^{6} \mathrm{~m}^{3}$ to $15.274 \times 10^{6} \mathrm{~m}^{3}$ or by $44.726 \times 10^{6} \mathrm{~m}^{3}$, due to the groundwater pumpage.

In the above way the magnitude by which the maximum "safe yield" of each polygon would increase or decrease was determined. A set of 27 values was thus obtained, and these were then transferred to computer cards. The initial set of 27 net deep percolation values, representing the present state, was replaced in the groundwater model by this new set of cards and a simulation run was made on the computer. This generated the annual water table elevations for each polygon over a time period of 10 years.
For each polygon a hydrograph was made by plotting the water table elevation against the time (Fig.36). These hydrographs then allowed the rate of change in water table elevation to be determined. The hydrograph of polygon 9 revealed that it would become waterlogged after 5 years. Similar problems can be expected in polygons 15,17 , and 18 , because in the 10 th year their water tables were found to be rising at a rate of 0.6 to 0.8 ma year, and their initial water







Fig. 36. Hydrographs of potygons $3,9,12,15,17$ and 18.
tables were already rather shallow.
In our first attempt to bring the water table under control, we adjusted the optimal water supply solution by imposing $9 \times 10^{6} \mathrm{~m}^{3}$ a year groundwater abstraction on polygon 9 and $12 \times 10^{6} \mathrm{~m}^{3}$ a year on polygon 15 . These adjustments were merely a matter of trial and error, but the groundwater model output often provides guidelines as to what and where changes in the water supply solution have to be made.

These adjustments were not yet satisfactory, because polygons 18 and 19 still revealed unacceptable water table changes, so further adjustments and new simulation runs were made to achieve a better control of the water table (Fig. 36). The adjustments also included the formation of two homogeneous clusters of polygons, one to be supplied with surface water, the other with groundwater.

The water supply solution, which was eventually obtained and accepted as being hydrodynamically feasible, is presented in Table 49 and Fig. 37. This figure shows that the polygons that will be supplied with surface water form a rather homogeneous block.

Figure 38 shows the cotal change in the polygons' water tables after 10 years. In general, the changes are moderate except that in the middle of the Plain a rise of about 8 to 9 m can be expected at the end of the $10^{\text {th }}$ year. From an agronomic viewpoint it is important to know at that depth the water table will be at the end of the $10^{\text {th }}$ year.This is shown in Fig. 39. No severe waterlogging problems are expected in the irrigated areas, with the exception of polygon 12 , which would need special attention because its water table would have risen to critical depths. Some additional groundwater pumpage could cure the situation, but no further adjustments were made here.

The rising water tables in the peripheral and unsupplied polygons are of little importance because these areas are either true salt deserts or do not otherwise enter into the solution.

### 10.1.3 Cost of water supply

Knowing the quantitics of water to be supplied (Table 49) and the $\mathrm{m}^{3}$ prices of surface water and well water, we could calculate the total cost of water supply for the adjusted solution. The results are shown in Table 50.

The cost of well water was Rls. $53,767,800$, and that of surface water Rls. $56,75!, 300$, a total of Rls. $110,519,100$. Since the total quantity of water

TABLE 49. Adjusted water supply solution of Scheme No. 1. MXRIV $=220 \times 10^{6} \mathrm{~m}^{3}$ a year, $\operatorname{MXSFY}=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and MXD $=$ RHS 1

| Polygon No, - | i 15XD | $\begin{aligned} \text { i PRD } \\ \text { ( } \end{aligned}$ |  | $\begin{aligned} & \text { i SRF (det) } \\ & \text { ) } \end{aligned}$ | i SRF (gross) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4.060 | 4.060 | - | 4.060 | 4.622 |
| 2 | 33.856 | - | - | - | - |
| 3 | 49.696 | 34.000 | 34.000 | - | - |
| 4 | 14.099 | 14.099 | - | 14.099 | 16.467 |
| 5 | 21.911 | 21.911 | - | 21.911 | 25.851 |
| 6 | 25.952 | 25.952 | - | 25.952 | 34.680 |
| 7 | 42.388 | 5.709 | - | 5.709 | 7.033 |
| 8 | 15.104 | - | - | . - | - |
| 9 | 23.874 | 23.874 | 8.000 | 15.874 | 20.081 |
| 10 | 24.430 | 24.430 | - | 24.430 | 30.116 |
| 11 | 22.929 | 22.929 | - | 22.929 | - 28.590 |
| 12 | 21.608 | 21.608 | 3.000 | 18.608 | 23.671 |
| 13 | 16.583 | 16.583 | 16.583 | - | - |
| 14 | 25.126 | 25.126 | - | 25.126 | 31.890 |
| 15 | 27.740 | 12.000. | 12.000 | - | - |
| 16 | 28.236 | - | - | - | - |
| 17 | 35.666 | - | - | - | - |
| 18 | 40.862 | 12.000 | 12.000 | - | - |
| 19 | 37.647 | 12.000 | 12.000 | - | - |
| 20 | 19.814 | - | - | - | - |
| 21 | 25.010 | - | - | - | - |
| 22 | 27.003 | 10.204 | 10.204 | - | - |
| 23 | 21.796 | - | - | - | - |
| 26 | 27.740 | - | - | - | - |
| total | 633.070 | 286.485 | 107.787 | 178.698 | 220.000 |

$100.0 \%$
$45.2 \pi$
$100.0 \%$
$37.6 \%$
$62.4 \%$


Fig. 37. Adjusted water supply solution of Scheme No.1.


Fig.38. Change in water table after 10 years. Adjusted solution of Scheme No. 1.


Fig. 39. Depth to water table after 10 years. Adjusted solution of Scheme No. 1 .
supplied was $286,485,000 \mathrm{~m}^{3}$, its average cost was $110,519,100 / 286,485,000=$ 0.3856 Rls per $\mathrm{m}^{3}$. The real average cost of water supply will be lower, because the canal system will have to be redesigned and its final length and dimensions will be much smaller than those used in the model. Consequently the investments and $m^{3}$ prices of surface water will be lower than those initially accepted in chis study.

TABLE 50. Calculation of the average water supply costs


### 10.1.4 Shadow prices of the used constraints

The obvious advantage of concentrating agricultural production'and water supply in a rather compact cluster of polygons in the upper and middle parts of the Plain is clearly demonstrated by the shadow prices obtained for the land constraints of the various polygons. Initially these shadow prices were calculated by the computer in terms of Rls per $\mathrm{m}^{3}$ of water, but we converted them by desk calculation into R1s per hectare. In this form they reflect the value of the last hectare of land in each of the supplied polygons.

The results of these calculations are given in Table 51 and Fig. 50. Polygons 12, 13, and 14, located in the middle of the Plain, had the highest shadow prices ( $16,000 \mathrm{Rls} / \mathrm{ha}$ ). Somewhat lower shadow prices ( $9,500 \mathrm{Rls} / \mathrm{ha}$ ) were found in polygons 4, 5, 6, 10, and 11 .

TABLE 51. Shadow prices of the water and land constraints in supplied polygons

| Polygon <br> No. | i MXD <br> $\left(\times 10^{6} \mathrm{~m}^{3}\right)$ | Shadow price <br> $\left(\mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}\right)$ | Water re- <br> quirement <br> $\left(\mathrm{m}^{3} / \mathrm{ha}\right)$ | Shadow price |
| :---: | :---: | :---: | :---: | :---: |



Eig.40. Shadow prices of the land constraints in supplied polygons.
10.1.5 The water supply solution and land allocation policies In the adjusted water supply solution under discussion, we assumed that the available water would be supplied only to the best land (Class I and Class II). It was interesting to find out how many hectares of this land would be irrigated,how many hectares would not,and where these lands are located.This information was obtained by dividing the values of iMXD and iPRD in Table 49 by the average water demand values of each polygon given in Table 38. The calculation gave the total area of Class I and Class II land and the area of this land that can be irrigated with the available water. The results are presented in Table 52. It appears that only 25,842 ha or 45 per cent of the total area of Cl ass I and

TABLE 52. Irrigated and non-irrigated areas of Class I and II land

| Polygon No. | Class I and II land ( ha ) |  |  | 2 of non-irrigated land |
| :---: | :---: | :---: | :---: | :---: |
|  | total | imrigated | non-irrigated |  |
| 1 | 338 | 338 | - | - |
| 2 | 3,127 | - | 3,127 | 100.0 |
| 3 | 4,590 | 3,140 | 1,450 | 31.6 |
| 4 | 1,260 . | 1,260 | - | . - |
| 5 | 1,957 | 1,957 | - | - |
| 6 | 2,318 | 2,318 | - | - |
| 7 | 3,915 | 527 | 3,388 | 86.5 |
| 8 | 1,395 | - | 1,395 | 100.0 |
| 9 | 2,205 | 2,205 | - . | - |
| 10 | 2,182 | 2,182 | - | - |
| 11 | 2,048 | 2,048 | - | - |
| 12 | 1,935 | 1,935 | - | ' - |
| 13 | 1,485 | 1,485 | - - | - |
| 14 | 2,250 | 2,250 | - | - |
| 15 | 2,520 | 1,090 | 1,430 | 56.7 |
| 16 | 2,565 | - | 2,565 | 100.0 |
| 17 | 3,240 | - | 3.240 | 100.0 |
| 18 | 3.712 | 1,090 | 2,622 | 70.6 |
| 19 | 3,420 | 1,090 | 2,330 | 68.1 |
| 20 | 1,800 | - | 1,800 | 100.0 |
| 21 | 2,272 | - | 2,272 | 100.0 |
| - 22 | 2,453 | 927 | 1,526 | 62.2 |
| 23 | 1,980 | - | 1,980 | 100.0 |
| 26 | 2,520 | - | 2,520 | 100.0 |
| TOTAL | 57,437 | 25,842 | 31,645 |  |
| 3 | 100.00 | 44.95 | 55.05 |  |

Class II land can be irrigated, which clearly demonstrates the scarcity of water compared with good quality land.
A further analysis of the water supply solution resulted in two different policies of land allocation, both of which are shown in Table 53. In Policy I we assumed, in line with Government objectives, an equitable distribution of irrigated 1 and between farmers, i.e. a farm size of 3.85 ha , as defined in alternative RHS 2. (The farms in polygons 1,10 to 15 , and 22 are slightly smaller.)

TABLE 53. Alternative policies of land allocation

| Polygon No. | Tocal number of farmers | Irrigated Class I \& Class II land <br> (ha) | Policy 1 |  | $\frac{\text { Policy }}{\text { Farm size }}$ <br> (ha) | II <br> Number of farmers that could be supplied |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Farm.size for RHS 2 <br> (ha) | Number of farmers that could be supplied |  |  |
| 1 | 106 | 338 | 3.19 | 106 | 3.19 | 106 |
| 2 | 469 | - | 3.85 | - | - | - |
| 3 | 493 | 3,140 | 3.85 | 816 | 6.37 | 493 |
| 4 | 93 | 1,260 | 3.85 | 327 | 13.55 | 93 |
| 5 | 338 | 1,957 | 3.85 | 508 | 5.79 | 338 |
| 6 | 369 | 2,318 | 3.85 | 602 | 6.28 | 369 |
| 7 | 369 | 527 | 3.85 | 137 | 1.43 | 369 |
| 8 | 196 | - | 3.85 | - | - | - |
| 9 | 326 | 2,205 | 3.85 | 573 | 6.76 | 326 |
| 10 | 267 | 2,182 | 3.50 | 623 | 8.17 | 267 |
| 11 | 322 | 2,048 | 3.50 | 585 | 6.36 | 322 |
| 12 | 470 | 1,935 | 3.50 | 553 | 4.12 | 470 |
| 13 | 315 | 1,485 | 3.50 | 424 | 4.71 | 315 |
| 14 | 430 | 2,250 | 3.50 | 643 | 5.23 | 430 |
| 15 | 362 | 1,090 | 3.70 | 295 | 3.01 | 362 |
| 16 | 129 | - | 3.85 | - | - | - - |
| 17 | 329 | - | 3.85 | - | - | - |
| 18 | 374 | 1,090 | 3.85 | 283 | 2.91 | 374 |
| 19 | 456 | 1,090 | 3.85 | 283 | 2.39 | 456 |
| 20 | 268 | - | 3.85 | - | - | - |
| 21 | 245 | - | 3.85 | - | - | - |
| 22 | 652 | 927 | 3.76 | 247 | 1.42 | 652 |
| 23 | 145 | - | 3.85 | - | - | - |
| 24 | 78 | - | - | - | - | - |
| 25 | 54 | * | - | - | - | * |
| 26 | 274 | - | 3.85 | - | - | - |
| 27 | 56. | - | - | - - | * | - |
| TOTAL | 7,986 | 25,842 |  | 7,005 |  | 5,742 |

With these farm sizes and the given area of irrigated 1 and, we found for each polygon the number of farmers that could be supplied with water (Column 5 in Table 53). These totalled 7,005. It can also be seen from the table that with Policy I all 106 farmers in polygon 1 would receive water, but that the 469 farmers in polygon 2 would receive none at all. In polygon 3 all 493 farmers would be supplied with water and there would be a surplus that could supply another 323 farmers. In polygons 4, 5, and 6 there would be sufficient water for an additional 234, 170, and 233 farmers above the actual number of farmers living in these polygons. In polygon 7 only 137 of the 369 farmers would be supplied, leaving 232 farmers unsupplied.

In this way it was found that with Policy I the total number of farmers that would receive water is 4,774 , the number of farmers that would not receive water is 3,212 , and that in polygons 3 to 6 and 9 to 14 there would be sufficient surplus water to supply another 2,231 farmers. Thus 2,231 farmers from the unsupplied peripheral polygons could be resettled in polygons. 3 to 6 and 9 to 14 , leaving 3,212-2,231 = 981 farmers unsupplied. For them other employment must be found. If, however, the size of the farms were to be 3.24 ha instead of the initially assumed 3.85 ha, all these 981 farmers could also be resettled in the supplied polygons. With either alternative, however, a quite extensive resettlement programe would be involved: 2,231 farmers with the first alternative and 3,212 farmers with the second.

Policy II consists of supplying water to the same polygons as in Policy I, but now to all the farmers living in these polygons. Table 53 indicates the impications of this policy. It can be seen that the farm sizes vary considerably, from !. 42 to 13.55 ha. Apart from the relatively large differences in farm size, which means great disparities in farmers' incomes, the policy also provokes a severe employment problem: Of the 7,986 farmers currently living in the Plain, only 5,742 would be supplied with water, leaving 2,244 farmers for whom other employment must be found. This problem is more severe than that of Policy I, where only 981 farmers or none at all would have to find other employment. From a socio-economic viewpoint, Policy I has the advantages of providing a more equitable distribution of land, water, and farmers' income, and does not pose a very severe employment problem.
10.1.6 Economic consequences of the hydrological adjustments

The initial water supply solution obtained from the linear progranming model is a unique solution, i.e. a solution that is optimal in an economic sense: the maximurn revenue is obtained from the available inputs.

The solution that was eventually accepted as feasible is not economically optimal because several adjustments had. to be made to bring the water table under control. Since these adjustments consisted of a certain groundwater abstraction from a polygon whose initial water demand had been met solely by surface water, the maximum revenue was reduced because this groundwater is costlier than surface water; the solution is no longer optimal.

The adjustments reduced the value of the objective function (net benefit) from Rls. $615,076,900$ to Rls : $606,330,900$, or by Rls. $8,746,000$. This reduction in the net benefit is the price that has to be paid to prevent certain parts of the plain from becoming waterlogged after 5 to 10 years, the other alternative being an artificial drainage system. The cost of a drainage system was not calculated, but it can be assumed to be much more than the above R1s. 8.746 million.

### 10.1.7 Summary

The main points of the adjusted water supply solution can be summarized as Eollows:

| Total area of Class 1 and Class II land | 57,487 | ha |
| :---: | :---: | :---: |
| Total irrígated area of Class $I$ and Clasa II land | 25,842 | ha |
| Total non-irrigated area of Class I and Class IT Iand | . 31,645 | ha |
| Total quantity of surface water supplied per year | 178,700,000 | $\mathrm{m}^{3}$ |
| Total quantity of groundwater supplied per year | 107,800,000 | $\mathrm{m}^{3}$ |
| Total quantity of water supplied per year | 286,485,000 | $\mathrm{m}^{3}$ |
| Total net benefit | 606,330,900 | R1s |
| Average cost of supplied water | 0.3856 | $18 / m^{3}$ |
| Number of existing farmers | 7.986 | (100\%) |
| Number of supplied fanmers | 4,774 | (60\%) |
| Number of unsupplied farmers | 3,212 | (40\%) |

### 10.2 Water supply scheme No. 2

### 10.2.1 Solution

For this scheme the annual river discharge was fixed at MXRIV $=150 \times 10^{6} \mathrm{~m}$, and the maximum water demand defined as RHS 1. To maintain the same area under irrigation as in the previous scheme, i.e. 25,842 ha of Class I and Class II land, the constraint maximum "safe yield" of the basin was released: the shortage of river water could be compensated for by groundwater pumpage above the annual $60 \times 10^{6} \mathrm{~m}^{3}$ that could be withdrawn in the previous scheme. The water supply solution obtained from the linear programing model is given in Table 54 and Fig. 41 .



Fig.41. Water supply solution of Scheme No. 2.

If we compare the solution of this scheme with that of the previous one, some differences in the areal distribution of groundwater pumpage are evident. These differences are sumazized in Table 55 , which indicates that the water deficiency of $56 \times 10^{6} \mathrm{~m}^{3}$ per year would be compensated for by groundwater pumpage in polygons 7, 10, 11 , and 14. To maintain the area of 25,842 ha under irrigated agriculture in years when only $150 \times 10^{6} \mathrm{~m}^{3}$ of surface water is available, a total annual groundwater pumpage of $163,823,000 \mathrm{~m}^{3}$ would be necessary. of the irrigation water supplied, 57 per, cent is well water and 43 per cent surface water. The last column of Table 54 shows the gross quantities of surface water needed to meet the demands of those polygons supplied with that water.

The total net benefit of the solution was found to be Rls. $597,772,500$.

TABLE 55. Comparison of polygonal groundwater abstractions for two different river discharges and mining and no mining of groundwater

| No. of scheme | 1 | 2 |
| :---: | :---: | :---: |
| MXRIV | $220 \times 10^{6} \mathrm{~m}^{3}$ | $150 \times 10^{6} \mathrm{~m}^{3}$ |
| MXSFY | $60 \times 10^{6} \mathrm{~m}^{3}$ | $\times 60 \times 10^{6} \mathrm{~m}^{3}$ |
| PolygonNo. | Groundwater abstraction |  |
|  | i WEL | i WEL |
| 3 | 34,000,000 | 34,000,000 |
| 7 | - | 5,709,000 |
| 9 | 8,000,000 | 8,000,000 |
| 10 | - | 24,430,000 |
| 11 | - | 771,000 |
| 12 | 3,000,000 | 3,000,000 |
| 13 | 16,583,000 | 16,583,000 |
| 14 | - | 25,126,000 |
| 15 | 12,000,000 | 12,000,000 |
| 18 | 12,000,000 | 12,000,000 |
| 19 | 12,000,000 | 12,000,000 |
| 22 | 10,204,000 | 10,204,000 |
| tetal | 107,787,000 $\mathrm{m}^{3}$ | 163,823,000 |

### 10.2.2 Testing the solution for its technical feasibility

To investigate the effect of the rather heavy groundwater pumpage on the water table, we simulated the water supply solution of Table 54 on the groundwater model for a period of 10 consecutive years.Figure 42 shows the change in water table for each polygon at the end of this period.For a major part of the plain a general decline of the water table was found, the largest drop being in the abstraction areas where at the end of the $10^{\text {th }}$ year the water table would have dropped 15 to 20 m. Polygon 12 was interesting as its water table remained rather stable. Pumping from this polygon, which at the time of our study was also an area of heavy abstraction, could be increased by some million $\mathrm{m}^{3}$. The surface water which would thus become available for use elsewhere could be supplied to polygon 14, where the water table decline is the greatest. Figure 43 shows the depth to the water table at the end of the 10 year period. In the abstraction centres the water table had dropped to a depth of 25 to 37 m below the ground surface, which is not excessively deep.

These two figures provide ample evidence that continuous mining of the groundwater resources at a rate of about twice the maximum "safe yield" of the basin is quite possible for a number of years, say 5 to 10 . In the perípheral polygons the water table remained rather stable, so that we do not have to fear the intrusion of salty groundwater.

### 10.2.3 Cost of water supply

As could be expected, mining of groundwater raises the average water supply cost (see Table 56).

The total cost of well water and surface water was Rls.118,791,600. The total net quantity of water supplied was $286,485,000 \mathrm{~m}^{3}$, so that the average water supply cost was $118,791,600 / 286,485,000=0.415 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$. This is slightly higher than the average supply cost of scheme No.l ( $0.3856 \mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}$ ), where no mining of the groundwater resources was permitted.

It should be recalled that lowering the water table by heavy pumping will raise the $m^{3}$ price of well water because more energy is required to lift the water to the surface. This means that the actual average water supply cost will be higher than the $0.415 \mathrm{Rls} / \mathrm{m}^{3}$ calculated.


Pig. 42. Change in water table after 10 years. Scheme No. 2.


Fig. 43. Depth to water table after 10 years. Scheme No. 2.

TABLE 56. Calculation of the average water supply costs of Scheme No. 2.

| Polygon No. | i WEL$\text { (million } n^{3} \text { ) }$ | $\begin{gathered} \operatorname{Cos} t \\ \text { i WEL } \\ \left(R L s / m^{3}\right) \end{gathered}$ | i SRF$\text { (million } m^{3} \text { ) }$ | $\begin{gathered} \text { Cost } \\ i \operatorname{sRF} \\ \left(R 1 s / m^{3}\right) \end{gathered}$ | Total cost |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | - WEL | i SRF |
|  |  |  |  |  |  | (R1s) |
| 1 | - | - | 4.622 | 0.1833 | - | 847,200 |
| 3 | 34.000 | 0.4593 | - | - | 15,616,200 | - |
| 4 | - | - | 16.467 | 0.2567 | - | 4,227,100 |
| 5 | - | - | 25.851 | 0.2599 | - | 6,718,700 |
| 6. | - | - | 31.680 | 0.2420 | - | 7,666,600 |
| '7 | 5.709 | 0.4990 | - | - | 2,848,000 | $-$ |
| 9 | 8.000 | 0.5082 | 20.081 | 0.2542 | 4,065,600 | 5,104,600 |
| 10 | 24.430 | 0.4592 | - | * | 11,818, 200 | - |
| 11 | 0.771 | 0.5053 | 27.628 | 0.2426 | 387,600 | 6,702,600 |
| 12 | 3.000 | 0.4593 | 23.67: | 0.2621 | 1,377,900 | $6,204,200$ |
| 13 | 16.583 | 0.5038 | - | - | 8,354,500 | - |
| 14 | 25.126 | 0.5213 | - | - | 13,098,200 | - |
| 15 | 12.000 | 0.5902 | - | - | 7,082,400 | - |
| 18 | 12.000 | 0.5653 | - | - | 6,783,600 | - |
| 19 | 12.000 | 0.5430 | . - | - | 6,135,600 | - |
| 22 | 10.204 | 0.4265 | - | - | 4,352,000 | - |
| TOTAL | 163.823 |  | 150.000 |  | 81,320,600 | 37,471,000 |

The main conclusion that can be drawn from this simulation is that during waterdeficient years the groundwater resources can be mined for a number of years, thus allowing the same level of agricultural production as in wetter years to be maintained ( 26,000 ha under cultivation).

Since with groundwater mining the average water supply costs are higher than with no groundwater mining, the total net benefit of this scheme would be Rls. $606,330,900-597,772,500=$ Rls. $8,558,400$ less than with Scheme No. 1.

### 10.3 Water supply scheme No. 3

### 10.3.1 Simulating river flow cycles

This scheme covered the simulation of a cycle of river flows, for which the river discharges of the two previously discussed schemes were used. A period of 40 years was considered, the first 16 years with an annual surface water availability of $220 \times .10^{6} \mathrm{~m}^{3}$ and no mining of the groundwater resources (NXSFY = $60 \times 10^{6} \mathrm{~m}^{3}$ per year), followed by 4 years with an annual surface water availability of $150 \times 10^{6} \mathrm{~m}^{3}$ and mining of the groundwatei to maintain the levet of agricultural production of the previous 16 years. This sequence was then repeated to make up the 40 years.

The groundwater model was used to simulate these alternating periods of different surface water supply and groundwater pumpage.

Figure 44 shows the change in water table at the end of the 40 years considered. A regional decline in the water table of 10 to 20 m can be expected, except in some of the peripheral polygons on the west and south.

The trend of the water table behaviour can also be seen in Fig. 45, which shows the hydrographs of polygons 4, 5, 10, and 14. Polygons 4 and 5 received river water only, but were affected by the pumpage in neighbouring polygons. The sharp drop in water tabie in' polygons 10 and 14 fron the $16^{\text {th }}$ to the $20^{\text {th }}$ year is due to the mining in those years of $24 \times 10^{6} \mathrm{~m}^{3}$ a year in polygon 10 and $25 \times 10^{6} \mathrm{~m}^{3}$ a year in polygon 14. At the end of the period of groundwater mining ( $21^{s t}$ year) the water table in those polygons started rising again, though did not reach its initial level. For polygons 10 and 14 the difference in water table between the $16^{\text {th }}$ and $36^{\text {th }}$ year is 6 m . During the following 4 years dry period, starting in the $37^{\text {th }}$ year, the water table dropped again sharply ( 14 to 17 m ). At the end of the $40^{\text {th }}$ year it was 6 to 7 m lower than at the end of the $20^{\text {th }}$ year.

Although the decline in water table was not so rapid that the solution must be rejected out of hand, it is obvious that the general trend of water table decline may cause problems in the long run.


Fig.44. Change in water table after 40 years: Scheme No.3.(River flow cyoling.)


Fig. 45 a. Hydrographs of potygons 4 and 5 over a period of 40 years. Scheme No. 3. (River flow cycling.)


Fig. $45 b$. Hydrographs of polygons 10 and 14 over a period of 40 years. Scheme No. 3 . (River flow cycling.)

### 10.4 Water supply scheme No. 4

10.4.1 Simulating artificial recharge

The generally declining water table observed in the previous scheme could, at least to some extent, be overcome by an artificial recharge of the groundwater basin. With the river flow constraints of $150 \times 10^{6} \mathrm{~m}^{3}$ and $220 \times 10^{6} \mathrm{~m}^{3}$ a year there are still substantial quantities of river water that could be used for this purpose. The most suitable sites for artificial recharge are the gravel fields in the head of the alluvial fan, covering major parts of polygons 1,4 , and 5. This scheme is basically the same as Scheme No.3, except that we simulated an artificial recharge of $8 \times 10^{6} \mathrm{~m}^{3}$ a year in each of these three polygons or a total of $24 \times 10^{6} \mathrm{~m}^{3}$ a year.

Figure 46 . shows the change in water table in the Plain after 40 years of this system's continuous operation. When this figure is compared with Fig. 44, the effect of the artificial recharge is clear.

At the end of the $40^{\text {th }}$ year the water table had dropped less than when no artificial recharge was applied. In the region with a falling water table, the drop was less than 10 m at the end of the $40^{\text {th }}$ year, except in polygons 6 and 18 , where a drop of 12 to 13 m was found.

The artificial recharge in the head of the alluvial fan had a stabilizing effect on the water table, as can also be seen in Fig.47: When pumping in polygons 10 and 14 stopped at the end of the $20^{\text {th }}$ year, the water table rose in the following years, though in this scheme too it did not reach its initial level. The difference, however, between the $16^{\text {th }}$ and $36^{\text {th }}$ years was only a few metres or less.

In general, because the changes in water table after 40 years are modest and will not cause special problems, we can conclude that this solution is feasible. of course, further refinements could be made to bring the water table under an even better control. Whenever available, slightly more than $24 \times 10^{6} \mathrm{~m}^{3}$ a year might be recharged into the upper polygons.Any available excess river water might be diverted to polygons $6, \ldots 14$, and 18 , where it can be used for infiltration and will thus prevent the water table from dropping too much. Similar minor corrective measures might be taken in polygons 12 and 13 , where somewhat more groundwater abstraction will prevent the water table from rising. In this study, however, we made no further attempts to improve matters.


Fig. 46. Change in water table after 40 years. Scheme No. 4. (River flow cyoling, recharge of 24 million $\mathrm{m}^{3}$ a year in polygons $\left.1,4,5\right)$.


Fig. 17. Hydrographs of poliggons 10 and 14 over a period of 40 years. Scheme No. 4. (River flow cycling, artificial recharge of 24 million $\mathrm{m}^{3}$ a year in polygons $2,4,5$. )

### 10.5 Water supply scheme No. 5

### 10.5.1 Solution

The aim of this scheme was to study the possible land and water use with a maximum river water availability of MXRIV $=340 \times 10^{6} \mathrm{~m}^{3}$ a year, a maximum groundwater pumpage of MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and a maximum water demand defined by MXD $=$ RHS 1.

From previous computer runs, we knew that certain polygons were sensitive to percolation from surface water irrigations, so to prevent them from becoming waterlogged, we imposed certain groundwater abstractions on them. For the scheme under discussion it did not make sense to repeat the whole sequence of computer runs, starting with the optimal solution, testing it with the groundwater model, making adjustments, testing it again, etc.

We therefore imposed merely on the linear programming model the condition that the water demand of polygons 1 to 15 be met. For polygons 3, 9, 12, 13, and 15 we imposed a minimum groundwater pumpage of $19.190,8.000,11.408,16.583$, and 12.000 miliion $\mathrm{m}^{3}$ a year and for polygon 8 a maximum agriculcurai production level of only $4.600 \mathrm{million} \mathrm{m}^{3}$. No agricultural production was permitted in the peripheral polygons $21,22,23$, and 26.

The solution produced by the linear programming model is shown in Table 57. and Fig. 48 . It can be seen that 64 per cent of the demand would be met. Of the water

TABLE 57. Water supply solution of scheme No. 5.
MXRIV $=340 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and MXD=RHS 1

| Polygon No. | - i MXD | $\begin{aligned} & \text { i PRD } \\ & \\ & \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \text { i SRF (net) } \\ & \text { ) } \end{aligned}$ | $\text { i } \mathrm{SRF} \text { (gross) }$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4.060 | 4.060 | - | 4.060 | 4.622 |
| 2 | 33.856 | 33.856 | - | 33.856 | 40.556 |
| 3 | 49.695 | 49.696 | 34.673 | 15.023 | 19.226 |
| 4 | 14.099 | 14.099 | - | 14.099 | 16.372 |
| 5 | 21.911 | 21.911 | - | 21.911 | 25.851 |
| 6 | 25.952 | 25.952 | - | 25.952 | 31.680 |
| 7 | 42.388 | 42,388 | - | 42.388 | 52.221 |
| 8 | 15.104 | 4.600 | - | 4.600 | 5.962 |
| 9 | 23.874 | 23.874 | 8.000 | 15.874 | 20.084 |
| 10 | 24.430 | 24.430 | - | 24.430 | 30.116 |
| 11 | 22.929 | 22.929 | - | 22.929 | 28.590 |
| 12 | 21.608 | 21.608 | 11.408 | 10.200 | 12.975 |
| 13 | 16.583 | 16.583 | 16.583 | - | - |
| 14 | 25.126 | 25.126 | - | 25.126 | 31.890 |
| 15 | 27.740 | 27.740 | 12.000. | 15.740 | 19.859 |
| 16 | 28.236 | - | - | - | - |
| 17 | 35.666 | 15.000 | 15.000 | - | - |
| 18 | 40.862 | 7.000 | 7.000 | - | - |
| 19 | 37.647 | 20.617 | 20.617 | - | - |
| 20 | 19.814 | 5.000 | 5.000 | - | - |
| 21 | 25.010 | - | - | - | - |
| 22 | 27.003 | - | - | - | - |
| 23. | 21.796 | - | - | - | - |
| 26 | 27.140 | - | - | - | - |
| total | 633.070 | 406.469 | 130.281 | 276.188 | 340.001 |
| 100.00\% |  | 64, 2\% |  | 68.0\% | . |
|  |  | 100.07 | 32.07 |  |  |



Fig.48. Water supply solution of scheme No. 5.
supplied, 32 per cent is well water and 68 per cent surface water. A total of 36,883 ha would be under irrigation.

The last column of Table 57 shows the gross quantity of surface water supplied. As was explained earlier, we calculated the quantity that each polygon supplied with surface water would contribute to the maximum "safe yield" of the basin. The total of these quantities was added to the maximum permissible pumpage of $60 \times 10^{6} \mathrm{~m}^{3}$ a year to give the actual safe yield of the basin: $117.253 \times 10^{6} \mathrm{~m}^{3}$ a year.

Obviously the solution obtained is not realistic as we cannot accept the 60 per cent risk that the surface water quantity of $340 \times 10^{6} \mathrm{~m}^{3}$ a year will not be available.

To maintain the area of 36,883 ha under irrigation in water-deficient years, more groundwater might be withdrawn from the basin to make up for the shortage of surface water. This problem will be examined below.

### 10.6 Water supply scheme No. 6

10.6.1 Solution

This scheme is a follow-up to Scheme No. 5 and was merely intended to examine the possibilities of increased groundwater pumpage in water-deficient years, i.e. in years when the flow is less than $340 \times 10^{6} \mathrm{~m}^{3}$ a year, but more than $220 \times 10^{6} \mathrm{~m}^{3}$. To mainctain the same level of agricultural production as in Scheme No. 5 , a total irrigated area of 36,883 ha, the groundwater must be mined. Thus the constraint MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year was released and as much groundwater could be pumped frow the basin as was neded to make up for the shortage of surface water.

The maximum quantity of surface water available was set at MXRIV $=220 \times 10^{6} \mathrm{~m}^{3}$ a year and the water demand as before: MXD $=$ RHS 1. Compared with Scheme No. 5 , there is $340-220=120 \times 10^{6} \mathrm{~m}^{3}$ less surface water available, and this must be withdrawn from the basin in addition to the permissible pumpage of $60 \times 10^{6} \mathrm{~m}^{3}$ a year.

The water supply solution obtained from the linear programming model for the above conditions is shown in Table 58 and Fig.49. It can be seen from the table that of the total quantity of irrigation water supplied, 55.4 per cent was well water and 44.6 per cent surface water.

This solution is not realistic either as we cannot accept the 20 per cent risk that the surface water quantity of $220 \times 10^{6} \mathrm{~m}^{3}$ a year will not be available. To maintain the same area of 36,883 ha under irrigation in years when the available surface water is less than $220 \times 10^{6} \mathrm{~m}^{3}$ a year, the groundwater resources might be used to make up for the water shortage.

TABLE 58. Water supply solution of Scheme No.6.
MXRIV $=220 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY is released, and $\mathrm{NXD}=$ RHS 1 .


| $100.00 \%$ | $64.2 \%$ |  |  |
| ---: | ---: | ---: | ---: |
|  | $100.0 \%$ | $55.4 \%$ | $44.6 \%$ |



Fig.49. Water supply solution of Scheme No.6.

### 10.7 Water supply scheme No. 7

### 10.7.1 Solution

As was mentioned above, in 4 years out of 20 the river flow will be less than $220 \times 10^{6} \mathrm{~m}^{3}$ per year, and in 1 year of these 4 the flow will be less than $150 \times 10^{6} \mathrm{~m}^{3}$, which is considered an acceptable risk. Inis scheme was intended to calculate the water supply solution for drought conditions, when only $150 \times 10^{6} \mathrm{~m}^{3}$ surface water a year would be available, and assuming the same level of agricultural production having to be maintained (PRD $=406.469$ million $\mathrm{m}^{3}$, corresponding with 36,883 ha of irrigated land).

Similar to the previous scheme, the safe yield constraint of the basin (MXSFY = $60 \times 10^{6} \mathrm{~m}^{3}$ a year) was released and as much groundwater could be pumped as was necessary to make up for the surface water deficiency of $340-150=190 \times 10^{6} \mathrm{~m}^{3}$ a year.

The water supply solution for the above conditions was calculated with the linear programing model and the results are presented ins.Table 59 and Fig. 50. The table shows that now 69.3 per cent of the irrigation water supplied is well water and 30.7 per cent surface water.

TABLE 59, Water supply solution of scheme No. 7. MXRIV $=150 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY is released, and $\mathrm{MXD}=$ RHS 1

| Polygon No. | i MXD | i PRD |  | $\begin{array}{r} i \text { WEL } \\ 10 \mathrm{i} 0 \mathrm{n} \end{array}$ | $\left.m^{3}\right)^{i \operatorname{SRF} \text { (net) }}$ | i SRF (gross) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 4.060 | 4.060 |  | - | 4.060 | 4.622 |
| 2 | 33.856 | 33.856 |  | - | 33.856 | 40.556 |
| 3 | 49.696 | 49.696 |  | 49.696 | - | - |
| 4 | 14.099 | 14.099 |  | - | 14.099 | 16.467 |
| 5 | 21.931 | 21.311 |  | - | 21.911 | 25.851 |
| 6 | 25.952 | 25.952 |  | - | 25.952 | 31.680 |
| 7 | 42.388 | 42.388 |  | 22.207 | 20.181 | 24,862 |
| 8 | 15.104 | 4.600 . |  | - | 4.600 | 5.962 |
| 9 | 23.874 | 23.874 |  | 23.874 | - | - |
| 130 | 24.430 | 24.430 |  | 24.430 | - | - |
| 11 | 22.929 | 22.929 |  | 22.929 | - | - |
| 12 | 21.608 | 21.608 | - | 21.608 | - | - |
| 13 | 16.583 | 16.583 |  | \$6.583 | - | - |
| 14 | 25.126 | 25.126 |  | 25.126 | - | - |
| 15 | 27.740 | 27.740 |  | 27.740 | - | - |
| 16 | 28.236 | - |  | - | - | - |
| 17 | 35.666 | 15.000 |  | 15.000 | - | - |
| 18 | 40.862 | 7.000 |  | 7.000 | * | - |
| 19 | 37.647 | 20.617 |  | 20.617 | - | - |
| 20 | 19.814 | 5.000 |  | 5.000 | - | - |
| 21 | 25.010 . | - |  | - | - | - |
| 22 | 27.003 | - |  | - | - | - |
| 23 | 21.796 | - |  | - | - | - |
| 26 | 27.740 | - |  | - | - | - |
| TOTAL | 633.070 | 406.469 |  | 281.810 | 124.659 | 150.000 |


| $100.00 \%$ | 64.27 |  |  |
| ---: | ---: | ---: | ---: |
|  | 100.08 | 69.37 | $.30 .7 \%$ |



Fig. 50. Water supply solution of Scheme No.7.

### 10.8 Water supply scheme No. 8

### 10.8.1 Simulating river flow cycles

For all the above schemes, the net deep percolation values associated with the water supply solutions were derived from the linear programming output. These values allow cycles of consecutive wet, normal, and dry years, and their effect on the water table, to be studied. The aim of this scheme was to simulate cycles of different river flows and groundwater pumpage,with and without mining of that resource.

As before, a simulation period of 40 years was chosen. The following cycle was arranged: 8 consecutive years with a river flow of $340 \times 10^{6} \mathrm{~m}^{3}$ a year and no mining of the groundwater resource, followed by 8 consecutive years with a river flow of $220 \times 10^{6} \mathrm{~m}^{3}$ a year and a moderate mining of the groundwater, and 4 consecutive years with a river flow of $150 \times 10^{6} \mathrm{~m}^{3}$ a year and more mining of the groundwater. This sequence was repeated to make up the 40 year period. The cycle was simulated by feeding the corresponding net deep percolation values of the polygons into the groundwater model, which then generated the annual water table elevations for each polygon over a period of 40 years.

Because of the high to very high pumping rates in the years when sufficient surface water would not be available to meet the water demand of the whole 36,883 ha of land under cultivation, we expected a considerable drop in the water table in major parts of the Plain. This was indeed found from the model, as can be seen from Figs. 51 and 52. Figure 51 shows the change in the water table after 20 years. In the northern and middle parts of the Plain a drop of 20 to 40 m was found at the end of the $20^{\text {th }}$ year, and in polygon 15 it was as much as 52 m . This unacceptable situation did not improve when a longer period was considered, as is shown in Fig. 52 . After 40 years, the water table in most of the Plain was found to have dropped 30 to 60 m , and in polygons 15 and 17 to 80 m . Of course some of the surface water in exces's of the assumed flows could be used to recharge the groundwater basin. But even if available, this water cannot be infiltrated into the soil in unlimited quantities. The fall in the water table is too great for artificial recharge to have a compensatory effect. Our only, but important, conclusion is therefore that an agricultural production level based on a surface water availability of $340 \times 10^{6} \mathrm{~m}^{3}$ a year and 36,883 ha under irrigated agriculture with groundwater mining in water-deficient years is not feasible. Such an agricultural production level can only be given serious consideration if the groundwater pumpage can be reduced to an acceptable amount by importing surface water from another catchment area or by conveying treated sewage water to the Plain from the nearby capital.


Fig.51. Change in water table after 20 years. Scheme No. 8 . (River flow cycling.)


Fig.52. Change in water table after 40 years. Scheme. No.8. (River flow cycling.)

### 10.9 Water supply scheme No. 9

### 10.9.1 Optimal solution

In the previous schemes we assumed a policy of supplying only the best quality land (Class I and Class It) optimally, disregarding the poorer lands currently being farmed. The water demand of that policy was designated as RHS 1.

We were also interested to know whether a solution could be obtained for a policy by which irrigation water could be supplied to all farmers currently living in the Plain, assuming that each farmer would possess 3.85 ha of good quality land. The water demand for this policy is designated as RHS 2 and the values for each polygon are given in Table 38 .

The scheme was defined as Eollows: maximum surface water availability: MXSRF = $220 \times 10^{6} \mathrm{~m}^{3}$ a year, maximum "safe yield" of the basin: MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and maximum water demand: $M X D=$ RHS 2.

The optimum solution obtained from the linear programming model is shown in Table 60 and Fig. 53. The fourth and fifth columns of the table show whether the water demand of a polygon would be met with well water or surface water or a combination of the two.

It can also be seen from this table that the demand of polygon 18 would only partly be met and that polygons 16,17 , and 26 would not receive any water from either source, because no more water was available.

In total there was an unsupplied demand of approximately $36 \times 10^{6} \mathrm{~m}^{3}$, or 11 per cent. Of the total quantity of water supplied, 38.1 per cent was well water and 61.9 per cent surface water.

The last colum of Table 60 shows the gross quantities of surface water released from the diversion weir to supply the various polygons. If we multiply these quantities with the corresponding coefficients of column 5 in Table 43, we obtain the quantity that each polygon supplied with surface water would contribute to the "safe yield" of the basit. The sum of these net percolations is $\mathbf{3 8 . 2 1 5 6}$ million $\mathrm{m}^{3}$ which, added to the "safe yield" of $60 \times 10^{6} \mathrm{~m}^{3}$, gives the actual safe yield of the basin, or 98.215 million $\mathrm{m}^{3}$. The total groundwater pumpage was 109.128 million $\mathrm{m}^{3}$. Since we assumed a return flow of 10 per cent for the well water, the net pumpage is therefore 90 per cent of 109.128 or 98.216 million $\mathrm{m}^{3}$, as calculated above.

TABLE 60. Optimal water supply solution of Scheme No.9. MXRIV $=220 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and MXD=RHS 2.



Fig.53. Optimal water supply solution of Scheme No.9.
10.9.2 Testing the optimal solution for its technical feasibility

From an economic viewpoint the water supply solution obtained is optimal, but it was still uncertain whether, if implemented, it would not create unacceptable water table changes in parts of the Plain. The solution was therefore tested by the groundwater model for the impact it would have on the water table.

To this aim the polygonal net deep percolation values that would occur if the optimal water supply scheme of Table 60 were implemented were calculated from the linear programning output, as was explained in Section 1.2. The set of 27 initial net deep percolation values (AQ-values) of the groundwater model, representing the present state, were replaced by this new set. The groundwater model simulated these new $A Q-v a l u e s$ for ten consecutive years and the computer generated the water table elevations for each polygon at the end of each year.

Table 61 shows the annual changes in water table for each polygon over the 10 year period, the total change in water table, and the depth to the water table in each polygon at the end of the $10^{\text {th }}$ year.

It can be seen from this table that polygons $9,12,16$, and 25 would become (nearly) waterlogged within 5 or 6 years, whereas in some of the pumped polygons $(19,20,21,22)$ the water table would drop 10 to 20 m in 10 years, due to the rather high abstraction rates.

The table also shows that the annual changes are greatest in the first years of the considered period and gradually diminish in the next years. In most of the polygons the water table.stabilizes after sone years, but in others (e.g. polygons $13,17,18,23$ ) it continues to change at a rather constant rate.

A situation where in some polygons the water table is steadily rising and in others steadily falling was, hydrodynamically speaking, not acceptable. The solution, though economically optimal, was not feasible from a technical viewpoint.

As a first attempt to bring the water table in these problem areas under control, a number of technical adjustments had to be made. By trial and error we adjusted the optimal water supply solution by imposing an annual groundwater abstraction of $8.0 \times 10^{6} \mathrm{~m}^{3}$ on polygon 9 , which had previously been supplied with surface. water only.

TABLE 61. Annual change in water table per polygon over a period of 10 years (in m) for Scheme No.9, after Eirst adjustment

| $\begin{aligned} & \text { Polygon } \\ & \text { No. } \end{aligned}$ |  |  |  | Y | $e$ | a | $r$ | $s$ |  |  | Total change | Total depth to water table |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
|  |  |  |  |  |  |  |  |  |  |  | after | 10 years |
| 1 | -2.1 | -1.4 | -0.9 | -0.6 | -0.5 | -0.3 | -0.3 | -0.2 | -0.2 | -0.2 | -6.7 | 95.2 |
| 2 | +0.4 | 0.0 | -0.1 | -0.2 | -0.1 | -0.2 | -0.1 | -0.2 | -0.1 | -0.2 | -0.8 | 44.8 |
| 3 | -1.3 | +0.1 | +0.4 | +0.2 | +0.2 | $+0.1$ | 0.0 | 0.0 | -0.1 | -0.1 | -0.5 | 11.5 |
| 4 | -0.6 | +0.2 | +0.2 | +0.1 | 0.0 | 0.0 | 0.0 | -0.1 | -0.1 | -0.1 | -0.4 | 33.9 |
| 5 | -0.5 | -0.2 | 0.0 | -0.1. | 0.0 | 0.0 | -0.1 | 0.0 | -0.1 | -0.1 | -1.4 | 35.1 |
| 6 | -2.9 | -2.1 | -1.5 | -1.1 | -0.8 | -0.6 | -0.4 | -0.4 | -0.3 | -0.2 | -10.3 | 44.8 |
| 7 | $+1.4$ | +0.6 | +0.2 | 40.1 | 0.0 | -0.1 | -0.2 | -0.1 | -0.2 | -0.1 | +1.6 | 11.4 |
| 8 | +0.7 | +0.8 | +0.8 | +0.6 | +0.8 | +0.7 | +0.6 | +0.5 | +0.4 | +0.4 | +6.5 | 13.5 |
| 9 | 44.3 | +3.4 | +2.7 | $+2.3$ | +1.9 | +0.4 | 0.0. | 0.0 | 0.0 | 0.0 | +15.0 | 0.0 |
| 10 | $+1.0$ | +1.1 | +1.0 | 40.9 | +0.7 | +0.4 | +0.2 | +0.1 | +0.1 | 0.0 | +5.5 | 4.5 |
| 11 | +4.2 | +2.7 | +1.2 | +0. B | +0.5 | +0.3 | +0.2 | +0.1 | 0.0 | 0.0 | +9.6 | 5.4 |
| 12 | +7.2 | +2.2 | +0.8 | +0.4 | +0.3 | +0.1 | +0.1 | +0.1 | 0.0 | 0.0 | +11.3 | 0.7 |
| 13 | -1.6 | -0.6 | -0.5 | -0.5 | -0.3 | -0.3 | -0.3 | -0.3 | -0.3 | -0.3 | -5.0 | 19.0 |
| 14 | +2.6 | +2.0 | +1.4 | $+1.0$ | +0.8 | $+0.5$ | +0.4 | +0.2 | +0.2 | +0.1 | +9.2 | 7.8 |
| 15 | +3.0 | $+2.8$ | +2.5 | $+2.2$ | +1.9 | $+1.5$ | $+1.3$ | +1.1 | +0.9 | +0.7 | +17.9 | 4.1 |
| 16 | 41.9 | +1.9 | $+2.0$ | $+1.9$ | +0.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | +8.0 | 0.0 |
| 17 | +0.7 | +0.9 | $+1.0$ | $+1.1$ | +1.2 | +1.1 | +1.1 | $+1.0$ | +0.9 | $+0.9$ | +9.9 | 1.1 |
| 18 | +2.7 | +2.2 | +1.6 | +1.4 | $+1.0$ | +0.9 | +0.7 | +0.5 | +0.5 | +0.4 | +11.9 | 3.1 |
| 19 | -2.5 | -2.0 | -1.6 | -1.2 | -1.0 | -0.7 | -0.7 | -0.5 | -0.4 | -0.4 | -11.0 | 28.0 |
| 20 | -6.7 | -3.8 | -2.4 | -1.8 | -1.3 | -1.1 | -0.9 | -0.7 | -0.7 | -0.6 | -20.0 | 40.0 |
| 21 | -3.7 | -3.6 | -3.0 | -2.4 | -1.9 | $-1.6$ | -1.4 | -1.1 | -1.0 | -0.8 | -20.5 | 31.5 |
| 22 | -4.7 | -3.4 | -2.6 | -2.0 | -1.8 | -1.4 | -1.2 | -1.1 | -0.9 | -0.8 | -19.9 | 29.9 |
| 23 | -0.5 | -0.6 | -0.6 | -0.6 | -0.6 | -0.6 | -0.5 | -0.4 | -0.5 | -0.5 | -5.6 | 8.6 |
| - 24 | -0.4 | -0.4 | -0.4 | -0.4 | -0.3 | -0.3 | -0.3 | -0.3 | -0.3 | -0.3 | -3.4 | 3.4 |
| 25 | +0.4 | +0.3 | +0.3 | +0.2 | +0.2 | +0.2 | +0.2 | +0.1 | +0.1 | 0.0 | +2.0 | 0.0 |
| 26 | +3.2 | +1.7 | +0.7 | +0.2 | -0.2 | -0.4 | -0.4 | -0.5 | -0.6 | -0.4. | +3.3 | 9.7 |
| 27 | +0.7 | +0.8 | +0.8 | +0.8 | +0.7 | +0.7 | +0.7 | +0.6 | +0.6 | +0.5 | +6.9 | 2.1 |

TABLE 62. Annual change in water table per polygon over a period of 10 years (in $m$ ) for Scheme No.9, after second adjustment.


Further adjustments were made in polygon 12 , where groundwater abscraction was increased to $10 \times 10^{6} \mathrm{~m}^{3}$ a year, in polygon 13 , where it was reduced to $10.5 \times 10^{6} \mathrm{~m}^{3}$ a year, and in polygon 17 , where $3.0 \times 10^{6} \mathrm{~m}^{3}$ a year pumpage was. imposed.

These adjustments were introduced into the optimal solution and a new linear programing computer run was made. This resulted in a (slightly) different water supply solution and somewhat different net deep percolation values for the polygons in question and some of their neighbours. This new set of 27 net deep percolation values were then fed into the groundwater model to examine their effect on the water table.

Although much better results were obtained than from the first computer run, polygon 17 was still causing problems. In a second attempt to bring the water table under better control, the groundwater abstraction in this polygon was raised from $3.0 \times 10^{6} \mathrm{~m}^{3}$ a year to $5.0 \times 10^{6} \mathrm{~m}^{3}$ a year and a new computer run on the groundwater model was made to examine the effect of this higher pumping rate.

Table 62 shows the annual change in water table over a period of 10 years and, as can be seen, the changes in the $10^{\text {th }}$ year have become small to very small in most polygons. The $2 \times 10^{6} \mathrm{~m}^{3}$ extra pumpage in polygon 17 had a marked effect on the water table: the total change in the 10 -year period was only 0.6 m instead of the 9.9 m in the previous run.

The effect of the adjustments on the water table in some of the problem poly gons can also be seen from the hydrographs of Fig. 54.

It is clear that further adjustments could be made to bring the water table in certain areas under even better control: in polygon 12 groundwater pumpage couid be increased by a few million $\mathrm{m}^{3}$ a year and in polygon 15 some pumpage could be introduced. Such adjustments were not made; it was assumed that the corrections discussed above would eventually yield a solution that was feasible:


Fig.54. Hydrographs of polygons 9, 12, 15, 17, and 18 over a period of 10 years. Scheme No.9.

Figure 55 shows the change in water table after 10 years. The highest rise in water table ( 8 to 11 m ) would occur $i_{n}$ the middle of the Plain (polygons li, 12 , 14,15 , and 16 ) and the greatest fall ( 12.5 m ) in polygon 22.

Figure 56 shows the depch to water table after 10 years of operating the adjusted water supply system. The map shows that nowhere in the cultivated area will the water table reach critical depths. Only in polygon 12 is the water table rather close to the land surface (about 3 m ). Some additional pumping of its groundwater could solve the problem.

The shallow water table in the peripheral polygons (Nos. $16,23,24,25,27$ ) does not pose special problems because most of these areas are salt desert and will remain so. Polygon 16 will not be supplied, with any water. Its rising groundwater can easily be drained off by the deeply incised Jaj Rud channel, which crosses the polygon. Its waterlogging problems may also be solved by sinking a number of wells in neighbouring upstream polygons to intercept a portion of the groundwater inflow into this polygon.

Remark

When evaluating the change in water table in the various polygons it should be borne in mind that the present situation is not a virginal one. There are already numerous deep wells in the Plain and certain of these have a high abstraction rate. This means that if our solution states that in an area of heavy pumping only a small quantity should be abstracted in the future, the groundwater simulation model may show that the water table will rise in such an area, whereas one would normally expect a fail. Hence, at first sight anomolous water table changes should not be ascribed to validity problems of the groundwater model.


Fig.55. Change in water tabte after 10 years. Adjusted solution of Scheme No.s.


Fig. 5b. Depth to water table after 10 years. Adjusted sotution of Scheme No.9.

### 10.9.3 Adjusted solution

After the above adjustments had been made, the water supply solution for Scheme 9 became that shown in Table 63 and in Fig. 57 . If we compare Tables 60 and 63 (the optimal and the adjusted solutions), we. find that the differences in quantities and percentages are small. The same can be said of a comparison of Figs. 53 and 57; surface water will be supplied to the northern and middle polygons and well water to the peripheral polygons.

From such a comparison it is clear that in many instances only minor adjustments are needed to reduce unacceptable water table changes to acceptable ones. It should, however, be borne in mind that any adjustment at a11, no matter how minor, means that the solution is no longer optimal; in other words, it reduces the value of the objective function (see Section 9.7).

### 10.9.4 Cost of water supply

By adjusting the optimal water supply solution of Scheme 9 , we raised the cost. of the undertaking, We were therefore interested to know the new. polygonal cost of water supply, the new total cost, and the new average water supply cost. Since the supplied quantities of surface water and groundwater per polygon were known (Table 63) and the $\mathrm{m}^{3}$ prices of the two comodities for each polygon were. also known, we were thus able to calculate the new costs (see Table 64).

It can be seen from this table that the total cost of well water was R1s. $52,806,800$ and that of surface water Rls. $62,217,200$. The sum of these quantities is R1s. $115,024,000$, representing the total cost of water supply. Since the total volume of water supplied was $286,204,000 \mathrm{~m}^{3}$, the average water cost was: $115,024 / 286,204=0.4019 \mathrm{Rls} / \mathrm{m}^{3}$.

In a similar way we could calculate the value of the objective function (or "net benefic"), viz. by multiplying the level of polygonal agricultural production (i PRD) by the polygonal cost ( $=$ net return per $\mathrm{m}^{3}$ ). The sum of these values minus the total cost of water supply gave the value of the objective function of the adjusted solution, which was found to be Ris. $536,001,300$.

## Remark

The term "net benefit" represents an aggregated net income of an imaginary ; association of Varamin farmers, to be distributed among them according to criteria of farm size, resource use, location, supply costs, soil fertility, etc. Its only meaning is for planning, because the planning engineer can use its value as an indicator of the economic profitability of the various alternative plans run on the computer. The absolute value of the "net benefit" is rather meaningLess as it does not include all the various additional development costs involved in such a large project, particularly those of infrastructure and resectlement.

TABLE 63. Adjusted water supply solution for Seheme No.9. MXRIV $=220 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and MXD=RHS 2.



Fig.57. Adjusted water supply solution of Scheme No.9.

TABLE 64. Calculation of the average water supply costs

| Polygon supplied No. | $\begin{aligned} & \text { Well water } \\ & \text { i WEL } \\ & \text { (million } \mathrm{m}^{3} \text { ) } \end{aligned}$ | Cost <br> i WEL $\left(\mathrm{Ri}_{\mathrm{s}} / \mathrm{m}^{3}\right.$ ) | $\begin{gathered} \text { Surface water } \\ \text { i SRF } \\ \text { (million m}{ }^{3} \text { ) } \end{gathered}$ | $\begin{aligned} & \text { Cost } \\ & \text { i SRF } \\ & \left(\mathrm{R} 1 \mathrm{~s} / \mathrm{m}^{3}\right) \end{aligned}$ | Total costs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | well water |  | rface water |
|  |  |  |  |  |  | (million | R1s) |
| 1 | - | - | 4.622 | 0.1833 | - |  | 0.8472 |
| 2 | - | - | 23.424 | 0.3058 | - |  | 7.1631 |
| 3 | 20.550 | 0.4593 | - | - | 9.4386 |  | - |
| 4 | - | - | 4.610 | 0.2567 | - |  | 1.1834 |
| 5 | - | - | 17.185 | 0.2599 | - |  | 4.4664 |
| 6 | - | - | 19.42 t | 0.2420 | - |  | 4.6999 |
| 3 | - | - | 18.954 | 0.2904 | - |  | 5.5042 |
| 8 | - | - | 10.595 | 0.3338 | - |  | 3.5366 |
| 9 | 8.000 | 0.5082 | 7.069 | 0.2542 | 4.0656 |  | 1.7696 |
| 10 | - | - | 12.904 | 0.2548 | - |  | 3.2879 |
| 11 | - | - | 15.733 | 0.2426 | - |  | 3.8168 |
| 12 | 10.000 | 0.4593 | 10.048 | 0.2621 | 4.5930 |  | 2.7908 |
| 13 | 10.553 | 0.5038 | 2.230 | 0.3193 | 5.3166 |  | 0.7120 |
| 14 | - | - | 21.330 | 0.2926 | - |  | 6.2412 |
| 15 | 3.000 | 0.5902 | 14.826 | 0.3016 | 1.7706 |  | 4.5471 |
| 17 | 5.000 | 0.6143 | 12.117 | 0.3167 | 3.0715 |  | 3.8375 |
| 18 | 5.000 | 0.5653 | 14.456 | 0.3172 | 2.8265 |  | 4.5854 |
| 19 | 10.517 | 0.5113 | 9.876 | 0.3241 | 5.3773 |  | 3.2008 |
| 21 | 10.381 | 0.4653 | - . | - | 4.8303 |  | - |
| 22 | 27.003 | 0.4265 | - | $\checkmark$. | 11.5168 |  | - |
| total | $110.004{ }^{\text {s }}$ |  | 220.000 |  | 52.8068 |  | 62,2172 |
| . | Total costs <br> Total water <br> Average cost | supplied | $\begin{array}{r} 115,024,000 \\ 286,204,000 \\ 0.4019 \end{array}$ | $\begin{aligned} & \mathrm{R1s} \mathrm{~s} \\ & \mathbf{a}^{3} \\ & \mathrm{Rl}_{\mathrm{s} / \mathrm{m}^{3}} \end{aligned}$ |  |  |  |

10.9.5 The, water supply solution and the irrigated area

Since our plan was based on the assumption that farmers will be supplied with water wherever they live in the Plain and that each farmer will possess 3.85 ha of land (except in some polygons where farms will have 3.2 , 3.5 , or 3.7 ha), it was interesting to know, for each polygon, the hectares of land that would be supplied with water, those that would not be supplied, and the number of farmers supplied and unsupplied. The results of these calculations are shown in rable 65. It can be seen from this table that only 26,066 ha (or 45 per cent) of the 57,487 ha of good land available will be irrigated and 31,421 ha (or 55 per cent) will not.

One of the most significant results of the solution, however, is that 6,981 farmers out of 7,986 (or 87.4 per cent) will be supplied with irrigation water and only 1,005 farmers (or 12.6 per cent) will not receive any water.

As far as palygon 20 is concerned, which was not included in the solution, the same remark can be made as in Section 10.1.

TABLE 65. Irrigated area, number of supplied and unsupplied farmers per polygon

| Polygon | Class I and If Iand (ha)' |  |  | Number of existing farmers |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | total | irrigated | non-irrigated | total | supplied | unsupplied |
| 1. | 338 | 338 | - | 106 | 106 | - |
| 2. | 3,127 | 1,806 | 1,321 | 469 | 469 | - - |
| 3. | 4,590 | 1,898 | 2,692 | 493 | 493 | - |
| 4. | 1,260 | 358 | - 902 | 93 | 93 | - |
| 5. | 1,957 | 1,301 | 656 | 338 | 338 | - |
| 6. | 2,318 | 1,421 | - 897 | 369 | 369 | - |
| 7. | 3,915. | 1,421 | - 2,494 | 369 | 369 | - |
| 8. | 1,395 | 755 | 640 | 196 | 196 | - |
| 9. | 2205 | 1255 | 950 | 326 | 326 | - |
| 10. | 2182 | 935 | 1,247 | 267 | 267 | - |
| 11. | 2,048 | 1,127 | 921 | 322 | 322 | - |
| 12. | 1,935 | 1,645 | 290 | 470 | 470 | - |
| 13. | 1,485 | 1,102 | 383 | 315 | 315 | - |
| 14. | 2,250 | 1,505 | 745 | 430 | 430 | - |
| 15. | 2,520 | 1,340 | 1,180 | 362 | 362 | - |
| 16. | 2,565 | - | 2,565 | 129 | - | 129 |
| 17. | 3,240 | 1,267 | 1,973 | 329 | 329 | - |
| 18. | 3.712 | 1,440 | 2,272 | 374 | 374 | - |
| 19. | 3,420 | 1,756 | 1,664 | 456 | 456 | - |
| 20. | 1,800 | - | 1,800 | 268 | $\sim$ | 268 |
| 21. | 2,272 | 943 | 1,329 | 245 | 245 | - |
| 22. | 2,453 | 2,453 | - | 652 | 652 | - |
| 23. | 1,980 | - | 1,980 | 146 | - | 146 |
| 24. | - | - | - | 78 | - | 78 |
| 25. | - | - | - | 54 | - . | 54 |
| 26. | 2,520 | - | 2,520 | 274 | - | 274 |
| 27. | - | - | - - | 56 | - | 56 |
| OTAL | 57,487 | 26,066 | 31,421 | 7,986 | 6,981 | 1,005 |
| 2 | 100.0 | 45.34 | 54.66 | 100.0 | 87.42 | 12.58 |

10.9.6 Shadow prices of farmers in the different polygons

The economy of water supply to the soils in the more central parts of the plain was calculated on the basis of their shadow prices. These shadow prices, which were presented in Rls per $\mathrm{m}^{3}$ in the linear programming output, were recalculated in Rls per farmer, thus reflecting the value of the last (marginal) farmer living in the middle of the Plain.

Table 66 shows the shadow prices for each polygon in Rl's per farmer, while Fig: 58 shows the distribution of these prices in map form. As can be seen, there is a great difference between the shadow prices of the middle polygons and those of the peripheral polygons. These differences clearly indicate the economic gains that would be possible if farmers from the peripheral areas were to move to the middle of the Plain, e.g. to polygons 12,13 , and 14 .



Fig.58. Polygonal shadow prices (in Rls per farmer).

### 10.9.7 Economic consequences of the hydrological adjustments

 The solution that was eventually accepted as feasible is not economically optimal because several adjustments had to be made to bring the water table under control. Since these adjustments consisted of a certain groundwater abstraction from a polygon whose water demand in the initial solution had been met solely by surface water; the value of the objective function (naxinum revenue or "net benefit") was reduced because this groundwater is costlier than surface water; the solution is no longer optimal.The adjustments made to obtain a feasible solution reduced the value of the objective function from R1s. $538,478,800$ to R1s. $536,001,300$, or by R1s.2,477,500. This reduction in the objective function is the price that has to be paid to prevent certain parts of the Plain from becoming waterlogged after some years, the other alternative being an artificial drainage system. The cost of a drainage system was not calculated, but it will be more than the above R1s.2,477,500.
10.9.8 Summary

The main points of the adjusted water supply solution can be summarized as follows:


## 10:10 Water supply scheme No. 10

### 10.10.1 Solution

From the statistical analysis of the surface water availability we found that in 4 years out of 20 the discharge of the river will be less than $220 \times 10^{6} \mathrm{~m}$ a year. It is evident that in these 4 years it will be impossible to meet the water demand of the 26,066 ha we were able to irrigate in Scheme No. 9 , unless we release the constraint Maximum "Safe Yield" of the groundwater basin (MXSFY = $60 \times 10^{6} \mathrm{~m}^{3}$ a year): This would allow us to mine the groundwater resources to a level that would satisfy the water demand of the 26,066 ha.

The aim of this scheme was merely to investigate the effect that groundwater mining would have on the water table in the Plain. For this purpose we cook the rather low river discharge of MXRIV $=150 \times 10^{6} \mathrm{~m}^{3}$ a year, or $70 \times 10^{6} \mathrm{~m}^{3}$ less than that used in Scheme No. 9 . The amount mined would therefore not be more than $70 \times 10^{6} \mathrm{~m}^{3}$ a year; in reality it will be less, because the percolation of surface water will add to the safe yield.

In this new scheme, we imposed the same agricultural production activity on the model as in Scheme No. 9 after the optimal water supply solution had been adjusted for a better water table control. This meant that we forced the computer to generate well water supplies in the same polygons as in Scheme No. 9 and surface water supplies in meigbbouring polygons, which thus form a single block as shown in Fig. 57.

The water supply solution obtained for the 26,066 ha is shown in Table 67 and Fig. 59.

Table 68 compares the results of Schemes 9 and 10 . With Scheme 10 the total groundwater pumpage was $163.6 \times 10^{6} \mathrm{~m}^{3}$ a year, or approximately $53.6 \times 10^{6} \mathrm{~m}^{3}$ a year more than with Scheme 9 . This additional quantity of groundwater was recovered from the polygons No. $12,13,14,17,18$, and 19.

TABLE 67. Water supply solution of Scheme No. 10
MXRIV $=150 \times 10^{6} \mathrm{~m}^{3}$ a year, MXSFY is released, and $M X D=$ RHS 2

| Polygon No. | i MxD |  |  |  | i SRF (net) | i SRF (gross) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | i 0 n | ) |  |
| 1 | 4.060 | 4.060 |  | - | 4.060 | 4.622 |
| 2 | 19.544 | 19.544 |  | - | 19.544 | 23.424 |
| 3 | 20.550 | 20.550 |  | 20.550 | - | - |
| 4 | 4.008 | 4.008 |  | - | 4.008 | 4.610 |
| 5 | 14.566 | 14.566 |  | - | 14.566 | 17.185 |
| 6 | 15.910 | 15.910 |  | - | 15.910 | 19.421 |
| 7 | $15.385^{\circ}$ | 15.385 |  | - | 15.385 | 18.954 |
| 8 | 8.174 | 8.174 |  | - | 8.174 | 10.595 |
| 9 | 13.588 | 13.588 |  | 8.000 | 5.588 | 7.069 |
| 10 | 10.468 | 10.468 |  | - | 10.468 | 12.904 |
| 11 | 12.618 | 12.618 |  | - | 12.618 | 15.733 |
| 12 | 18.370 | 18.370 |  | 18.370 | - | - |
| 13 | 12.306 | 12.306 |  | 12.306 | - | - |
| 14 | 16.806 | 16.806 |  | 16.289 | 0.517 | 0.657 |
| 15 | 14.751 | 14.751 |  | 3.000 | 11.751 | 14.826 |
| 16 | 5.471 | . - |  | - | - | - |
| 17 | 13.947 | 13.947 |  | 13.947 | - | * |
| 18 | 15.851 | 15.851 |  | 15.851 | - | - |
| 19 | 19.330 | 17.908 |  | 17.908 | - | - - |
| 20 | 11.360 | - |  | - | - | - |
| 21 | 10.381 | 10.381 |  | 10.381 | - | * |
| 22 | 27.003 | 27.003 |  | 27.003 | - | - |
| 23 | 6.186 | - |  | - | - | - |
| 26 | .11.613 | - |  | - | - | - |
| TOTAL | 322.256 | 286.204 |  | 163.605 | 122.599 | 150.000 |
|  | 100\% | $88.8 \%$ |  |  |  |  |
|  |  | $100 \%$ |  | 57.2\% | 42.87 | . |

TOTAi REVENUE:
R1s $528,847,500$


Fig.59. Water supply solution of Scheme No. 10.

TABLE 68. Comparison between groundwater supplies in situations of mining and no mining of the groundwater regources (million $\mathrm{ma}^{3}$ )

10.10.2 Testing the solution for its technical feasibility

To investigate the effect that the solution given in Table 67 would have on the water table, the net deep percolation values of the different polygons were derived from the linear programing output and fed into the groundwater model. The change in water table after 10 years is shown in Fig. 60 . No alarming water table changes were found except in a few polygons where the water table had dropped 20 m or more (polygons 17 to 22 , where most of the pumping took place). Figure $6 t$ shows the depth to the water table after 10 years of operation. It appears that the water table in the polygons where groundwater is abstracted would be 20 to 40 m below the ground surface at that time. As this is not exceptionally deep, the solution is therefore acceptable. It should be noted, however, that the $\mathrm{m}^{3}$ price of groundwater used for these polygons is no longer valid at such great depths. The main conclusion that can be drawn from the results obtained with this scheme is that for a number of consecutive years a certain mining of the groundwater resource is possible. Some of the peripheral polygons, however, would need special attention, because the rather sharp drop in their water tables may cause the salty groundwater from outside the basin to flow to these areas.
For the adjusted water supply solution of Scheme No. 9 we calculated a net benefit of Rls. 536.0 million , and for the solution of Scheme 10 R 1 s .528 .8 million , or only Rls. 7.0 million less.


Fig.60. Change in water table after 10 years. Scheme Mo. 10.


Fig.61. Depth to water table after 10 years. Scheme No. 10.

### 10.11 Water supply scheme No. 11'

10.11.1 Simulating river flow oycles

In the previous studies, operating with the "safe yield" concept and a maximum surface water availability of $220 \times 10^{6} \mathrm{~m}^{3}$ a year, we found a water supply solution for 26,000 ha that was technically feasible. We also found that a certain groundwater mining was possible in water-deficient years, to keep that area under cultivation.

In our stepwise approach to the water supply problem, the next step was to combine Schemes 9 and 10 to obtain periods of sufficient surface water alternating with periods of a surface water shortage during which the groundwater would be mined. More specifically, we considered a period of 40 years made up of 16 consecutive years when $220 \times 10^{6} \mathrm{~m}^{3}$ river water would be available annually, followed by 4 years with an annual water availability of $150 \times 10^{6} \mathrm{~m}^{3}$, after which the sequence was repeated to complete the 40 yeais.

The simulations on the groundwater model were made with the adjusted water supply solution obtained in Scheme No. 9.

Figure 62 shows the water table change's at the end of the 40 -year period. They were found to be rather modest. Only in polygons $6,20,21$, and 22 , can a drop in water table of some 15 to 18 m be expected. In the other polygons the drop will be less than 15 m , except in polygons $8,9,11,15$, and 16 , where it will rise a few metres.

It is clear that a situation where water tables are rising in some areas and dropping in others, is not a stable hydrodynamic situation.: This instability is clear from the hydrographs of the polygons depicted in Fig. 63. The hydrographs of polygons $13,14,17$, and 18 reflect the increased groundwater pumpage in the $16^{\text {th }}$ and following years. From the $21^{\text {st }}$ to the $36^{\text {th }}$ year there is again sufficient surface water available and pumping is reduced to the initial rate of $60 \times 10^{6} \mathrm{~m}^{3}$ a year. Consequently the water table starts rising. In polygons 13 and 14 the water table stabilizes at the end of each 16 -year period, though at a level that is several metres lower than the initial levels.

During the final 4 years the pumpage is again increased and we see the water table dropping sharply and to a lower level than in the preceding 4 -year period of mining. (For the rates of groundwater pumpage, see Table 68).


Fig.62. Change in water table after 40 years. Scheme No.11. (River flow cycling.)


Fig. 63a. Hydrographe of polygons 13 and 17 over a period of 40 years. Soheme Ho.11. (fiver flow oycling.)


Fig. 53b. Hydrographs of polygons 21 and 22 over a period of 40 years. Scheme No.11. (Fiver flow cycling.)


Fig. 63c. Hydrographs of polygons 14 and 18 over a period of 40 years. Scheme No.11. (River flow cycling.)

The pumpage in polygons 21 and 22 is constant over the whole 40 -year period, yet the effect of alternating mining and no mining of groundwater is clearly reflected in neighbouring polygons by the drop in their water tables in the $18^{\text {th }}$ and following years. In the next l6-year period the water table stabilizes, but at a lower level than before.

All the hydrographs of Fig. 63 show a general trend of a declining water table. Although the changes are quite modest, in the long run problems can be expected in certain parts of the Plain.

### 10.12 Water supply scheme No. 12

### 10.12.1 Simulating artificial recharge

In the previous schemes, we worked with constant river flows over simulation periods of 10 and 40 years. In reality, however, there are years when the river flow exceeds the ones we used in our simulations. In 1.6 years out of 40 , for instance, the flow exceeds $340 \times 10^{6} \mathrm{~m}^{3}$ a year and in another 16 years it is less than this amount but more than $220 \times 10^{6} \mathrm{~m}^{3}$. In 7 years the flow is less than $220 \times 10^{6} \mathrm{~m}^{3}$ but more than $150 \times 10^{6} \mathrm{~m}^{3}$, and in 1 year it is less than $150 \times 10^{6} \mathrm{~m}^{3}$. In the previous scheme the applied cycle of $220 \times 10^{6} \mathrm{~m}^{3}$ and $150 \times 10^{6} \mathrm{~m}^{3}$ a year. thus implies that considerable quantities of river water were not being used effectively and were simply disposed of to the desert through the main Jaj Rud branch on the west.

Since the simulation of the above river flow cycle revealed a general trend of water table decline due to groundwater mining in water-deficient years; we decided to use a portion of the excess river flow to simulate an artificial recharge of the groundwater basin. This water supply scheme is the same as Scheme No.ll, except that we imposed an artificial recharge of $8.0 \times 10^{6} \mathrm{~m}^{3}$ a year in polygons $1 ; 4$, and 5 , or a total of $24 \times 10^{6} \mathrm{~m}^{3}$ a year over a period of 40 years.

Figure 64 shows the change in water table after these 40 years. The effect of the artificial recharge is evident: the fall in water table in the northern polygons was less than without the recharge, and in adjacent polygons it even rose slightly. In polygons $8,9,10,11,15$, and 16 this rise was somewhat faster than without the recharge (compare Fig.62). A modest groundwater pumpage in some of these polygons could easily bring their water table under control.

Figure 65 showis a plot of the water table against time for the same polygons as in Fig. 63. The shapes of the graphs are almost identical, except that after the groundwater mining in the $17^{\text {th }}$ to the $20^{\text {th }}$ year the water table recovered almost completely. In these and all other polygons, the water table elevations in the $16^{\text {th }}$ and the $36^{\text {th }}$ year differed only slightly, varying from -2.4 to +1.5 m .

We may therefore conclude that a hydrodynamically feasible water supply solution that makes it possible to maintain 26,000 ha of land under irrigated agriculture has been found, Based on the water demand defined by RHS 2, the solution consists of a maximum river flow of $220 \times 10^{6} \mathrm{~m}^{3}$ a year, groundwater pumpage at a rate of $60 \times 10^{6} \mathrm{~m}^{3}$ a year, mining of this resource in surface water deficient years, and an annual artificial recharge of $24 \times 10^{6} \mathrm{~m}^{3}$.


Fig.64. Change in water table after 40 years. Scheme No.12. (River flow cycling and artificial recharge in polygons 1, 4, and 5).


Fig. 65a. Hydrographs of polygons 13 and 17 over a period of 40 years. Scheme No.12. (River flow cyoling and artificial recharge:)


Dif. 65 . Hyarographs of polygons 14 and 18 over a period of 40 years. Scheme No.12. (River flow cycling and artificial recharge.)

### 10.13 Parametric programming

### 10.13.1 Schemes Nos. 13 to 17

In the previous sections we worked with certain probabilities of surface water availability and arrived at two feasible water supply solutions. The importation of foreign water to the Plain was not considered in any of these schemes. There are, however, two possible sources of foreign water. One is the Lar Rud, which rises in the Elburz Mountains and runs to the Caspian Sea. A dam is under construction in this river and a portion of its water will be diverted to Teheran in the near future to meet the cown's ever-growing water demand. At the time of our study it was uncertain whether any water from the Lar Rud could be made available to the Varamin Plain and if so, in what quantities.

The other source of foreign water is treated sewage water from Teheran. With more and more water being conveyed to the town, the problem of its sewage water disposal may become acute. That water could be collected and treated, and a portion conveyed by canal or pipe line to the Varamin Plain. At the time of our study nothing definite could be said about this source of water.

Notwithstanding these uncertainties we wanted to examine the economic consequences of importing foreign water to the Plain. To do so, we applied parametric programing, which means that all the activities and constraints of a certain scheme are kept constant except one, viz the maximum river water availability, which is considered a variable. For this purpose the constraint MXRIV was increased by small increments of $30 \times 10^{6} \mathrm{~m}^{3}$.

The optimal water supply solutions and corresponding revenues were calculated for the following surface water constraints: MXRIV $=220,250,280,310$, and 340 million $m^{3}$ a year in combination with the "safe yield" concept (no mining of groundwater or MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year) and the economically more profitable water demand constraint defined by MXD $=$ RHS 1 .

By means of this parametric progranming we were able to indicate the expansion of the irrigated area as more and more surface water became available and the effect this would have on the value of the objective function.
Table 69 shows how the linear programming model distributed the four increasing quantities of surface water over the polygons. The quantities presented in the table are gross quantities to be released from the diversion weir to meet

TABLE 69. Parametric runs, expansion of surface water supply, MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and $\mathrm{MXD}=\mathrm{RHS} 1$

| Scheme No. | 13 | 14 | 15 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MXRIV | 220 | 250 | 280 | 310 | 340 |
|  | (million $\mathrm{m}^{3}$ ) |  |  |  |  |
| Polygon |  |  |  |  |  |
| 1 | 4.622 | 4.622 | 4.622 | 4.622 | 4.622 |
| 2 | - | - | 40.556 | 40.556 | 40.556 |
| 3 | - | - | - | - | - |
| 4 | 16.467 | 16.467 | 16.467 | 16.467 | 16.299 |
| 5 | 25.851 | 25.851 | 25.851 | 25.851 | 25.851 |
| 6 | 31.680 | 31.680 | 31.680 | 31.680 | 31.680 |
| 7 | - | 23.095 | 12.799 | 49.750 | 52.220 |
| 8 | - | - | - | - | - |
| 9 | 30.201 | 30.201 | 30.201 | 30.201 | 30.201 |
| 10 | 30.116 | 30.116 | 30.116 | 30.116 | 30.116 |
| 11 | 28.590 | 28.590 | 28.590 | 28.590 | 28.590 |
| 12 | 20.583 | 27.488 | 27.236 | 20.277 | 12.976 |
| 13 | - | - | - | - | - |
| 14 | $31.890^{\circ}$ | 31.890 | 31.890 | 31.890 | 31.890 |
| 15 | - | - | - | - | 34.999 |
| 16 | $\div$ | - | - | - | - |
| 17 | - | - | - | - | - |
| 18 | - | - | - | $\sim$ | - |
| 19 | - | - | - | - | - |
| 20 | - | - | - | - | - |
| 21 | - | - | - | - | - |
| 22 | - | - | - | - | - |
| 23 | - | - | - | - | - |
| 26 | - | - | - | - | - |
| total | 220.000 | 250.000 | 280:000 | 310.000 | 340.000 |

the water demand of the various polygons. It is interesting to note how the extra water is distributed over the polygons:if $250 \times 10^{6} \mathrm{~m}^{3}$ water is available instead of $220 \times 10^{6} \mathrm{~m}^{3}$, for instance, the extra quantity is supplied to polygons 7 and 12; if $280 \times 10^{6} \mathrm{~m}^{3}$ water is available instead of $250 \times 10^{6} \mathrm{~m}^{3}$, the extra water is supplied to polygon 2 whose maximum demand is entirely satisfied at the expense of the supplies to polygons 7 and 12 .

Table 70 shows the expansion of agricultural production over the polygons as more water becomes available. The corresponding revenues, irrigated area, number of farmers supplied, and "net income" per farmer were calculated, and were found to increase gradually as the surface water quantity increases.

TABLE 70. Parametric runs, expansion of agricultural production, $i$ PRD, MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year, and MXD $=$ RHS 1

| Scheme No. | 13 | 14 | 15 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MXRIV | 220 | 250 | . 280 | 310 | 340 |
|  |  | $\text { (million } m^{3} \text { ) }$ |  |  |  |
| Polygon |  |  |  |  |  |
|  | - |  |  |  |  |
| 1 | 4.060 | 4.060 | 4.060 | 4.060 | 4.060 |
| 2 | - | - | 33.856 | 33.856 | 33.856 |
| 3 | 49.696 | 49.696 | 49.696 | 49.696 | 49.696 |
| 4 | 14.099 | -14.099 | 14.099 | 14.099 | 14.099 |
| 5 | 21.911 | 21.911 | 21.911 | 21.911 | 21.911 |
| 6 | 25.952 | 25.952 | 25.952 | 25.952 | 25.952 |
| 7 | - | 18.747 | 10.383 | 40.382 | 42.388 |
| 8 | - | - | - | - | - |
| 9 | 23.874 | 23.874 | 23.874 | 23.874 | 23.874 |
| 10 | 24.430 | 24.430 | 24.430 | 24.430 | 24.430 |
| 11 | 22.929 | 22.929 | 22.929 | 22.929 | 22.929 |
| 12 | 21.608 | 21.608 | 21.608 | 21.608 | 21.608 |
| 13 | 16.583 | . 16.583 | 16.583 | 16.583. | 16.583 |
| 14 | 25.126 | 25.126 | 25.126 | 25.126 | 25.126 |
| 15 | * | - | - | - | 27.740 |
| 16 | - | - | - | - | - |
| 17 | - | - | - | - | - |
| 18 | - | - | - | - | - |
| 19 | - | $\sim$ | - | - | 0.307 |
| 20 | - | $\sim$ | - | - | - |
| 21 - | 6.200 | 20.396 | 25.010 | 25.910 | 25.010 |
| 22 | 27.003 | 27.003 | 27,003 | 27.003 | 27.003 |
| 23 | - | - | - | - . | - |
| 26 | - | $\sim$ | . ${ }^{+}$ | - | - |
| TOTAL | 283,471 | 316.414 | 346.520 | 376.519 | 406.572 |
| Total revenue (million Rls) | 615.077 | 655.262 | 669.860 | 744.446 | 788.277 |
| Irrigated area (ha) | 25,857 | 28,606 | 31,379 | 34,150 | 36,883 |
| Number of Earmers supplied | 7,069 | 7,820 | 7,986 | 7,986 | 7,986 |
| Average "net income" <br> (Rls/farmer) | 87,000 | 83,760 | 83,897 | 93,163 | 98,673 |

Figure 66 sumarizes the results obtained. As can be seen, only five more polygons will be wholly or partly included in the irrigated area, viz, the polygons $2,7,15,19$, and 21 . The peripheral polygons $8,16,17,23$, and 26 will remain unsupplied as will polygon 20, although this is merely a matter of its slightly higher well water price, compared with that of adjacent polygons.

The most important conclusion that can be drawn from the parametric programing is that the comparative advantage of having a compact and centrally located irrigated block is emphasized. This is an important point on which to base decisions about practical project implementation. If there is a chance of importing foreign water to the Plain, it may help the development authority to decide whether the marginal (peripheral) villages should be abandoned or not, and if so, how many of them? On this point parametric programming clearly indicates that only the polygons $2,7,15,19$, and 21 should be added to the irrigated area. The farmers living in those polygons then have a fair chance of receiving water at a later date and need not be resettled.

The above conclusions are, of course, based on the water demand assumption defined by RHS 1 (only the best land will be supplied), which implies a rigorous resettlement of farmers within the region.

Table 71 gives the shadow prices of the water constraints, total and average revenues per $\mathrm{m}^{3}$ of supplied water, and the percentages of surface water and well water supplied under the different schemes.

Figure 67 shows these results in the form of graphs.


TABLE 71. Results of parametric programming: MXRIV variable, MXD $=$ RHS 1 and MXSFY $=60 \times 10^{6} \mathrm{~m}^{3}$ a year

| HRRIV | Shadow price ( RL / $/ \mathrm{m}^{3}$ ) |  |  |
| :---: | :---: | :---: | :---: |
|  | surface watet | we 11 water |  |
| 220 | 1.489 | 1.551 |  |
| 250 | 1.487 | 1.551 |  |
| 280 | 1.486 | 1.548 |  |
| 310 | 1.486 | 1,548 | . |
| 340 | 1.443 | 1.500 |  |
|  | (b) Total pevenue and average ravenue perm $\mathrm{m}^{3}$ |  |  |
| EXRIV | Total revenue million Ris | Total supplied million $\mathrm{m}^{3}$ | Average revenue $\mathrm{Rl}_{\mathrm{s}} / \mathrm{m}^{3}$ |
| 220 | 615.1 | 283.5 | 2.17 |
| 250 | 655.2 | 316.4 | 2.07 |
| 280 | 699.9 | 346.5 | 2.02 |
| 310 | 744.4 | 376.5 | 1.98 |
| 340 | 788.3 | 406.6 | 1.94 |

(o) Composition of water supply

| MXRIV | Total supplied million $\mathrm{m}^{3}$ | Surface wacer |  | Well water |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | million ${ }^{3}$ | 3 | million m ${ }^{3}$ | 7 |
| 220 | 283.5 | 175.6 | 61.94 | 107.9 | 38.06 |
| 2501 | 316.4 | 202.7 | 64.06 | 113.7 | 35.94 |
| 280 | 346.5 | 228.0 | 65.80 | 118.5 | 34.20 |
| 310 | 376.5 | 252.5 | 67.07 | 124.0 | 32.93 |
| 340 | 406.6 | 276.6 | 68.03 | 130.0 | 31.97 |



Fig.67. Shadow prices of well water and surface water, average and total revenue for various surface water availabilities.

## 11. Adjusting the water supply solutions to overcome the monthly river discharge deficiencies

For reasons of simplicity and for lack of sufficient data, all calculations and computer runs were made on an annual basis. However, as the Farahnaz Pah1avi Dam at Latiyan regulates the flow of the Jaj Rud only to some extent, the question arose whether the river would, at all times, be able to meet the demand in those areas that are to be supplied with river water; in other words, how serious is the discrepancy between the monthly water demand and the monthly river discharge?

The answer to this question can be found from Table 72, which shows the monthly demand as finally accepted for the project and the monthly discharge of the Jaj Rud as an average of the 22 years of data obtained from the Darvazeh gauging station and shown in Table 6. It can be seen from Table 72 that there is a river water deficit.in the period June to November, with July and August as the most critical months. In these two months the water demand is 17.1 and 14.5 per cent of the total annual demand and the river discharge only 7.0 and 3.9 per cent of its total annual discharge, resulting in a water deficit of 10.1 and 10.6 per cent, respectively.

TABLE 72. Monthly irrigation water demand and river discharge

| Month | Water demand |  | Discharge of the Jaj Rud at Darvazein |  | Difference <br> \% |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | million m ${ }^{3}$ | \% | million $\mathrm{m}^{3}$ | $\%$ |  |
| October | 12.8 | 4.3 | 10.08 | 2.8 | - 1.5 |
| November | 10.0 | 3.3 | 12.87 | 3.5 | + 0.2 |
| December | 5.2 | 1.7 | 12.34 | 3.4 | $+1.7$ |
| January | 0.7 | 0.2 | 11.35 | 3.1 | + 2.9 |
| February | 0.6 | 0.2 | 14.18 | 3.9 | + 3.7 |
| March | 10.8 | 3.6 | 28.31 | 7.8 | + 4.2 |
| April | 36.2 | 12.0 | 68.22 | $18.8{ }^{\circ}$ | + 6.8 |
| May | 50.2 | 16.7 | 97.10 | 26.7 | +10.0 |
| June | 51.2 | 17.0 | 60.11 | 16.5 | - 0.5 |
| July | 51.3 | 17.1 | 25.35 | 7.0 | - 10.1 |
| Augus t | 43.5 | 14.5 | 14.09 | 3.9 | - 10.6 |
| September | 28.2 | 9.4 | 9.64 | 2.6 | - 6.8 |
| total | 300.7 | 100.0 | 363.64 | 100.0 |  |

In our efforts to overcome the problem of surface water shortage in the summer and early autumn, it was of course obvious that any solution found for the two most critical months would automatically solve the problem of the other waterdeficient months. In fact, we could even limit our considerations to the month of August when the river water deficiency is the greatest.

It was also obvious that we only had to examine the water shortage in those polygons that were to receive river water or a combination of river and well water; those that were to be supplied solely with well water would not be affected by the shortage of surface water.

If we refer back to the water supply solution obtained for the driest year we considered, i.e. the one with a maximum river water availability of $150 \times 10^{6} \mathrm{~m}^{3}$, combined with the water demand constraint defined by MXD = RHS 1 (see Table 54), we find that the polygons that would suffer from a shortage of surface water are Nos.1, 4, 5, 6, 9, 11, and 12. For these polygons, a supplementary water supply had to be found.

The only way to overcome a surface water deficiency is to draw upon the groundwater resources of the Plain. We then wanted to find out whether the existing well capacities were sufficient to meet the needs or whether additional welis would have to be sunk. How we went about finding the answers to these questions will be explained below.

The existing well capacity per polygon is shown in Table 73. In preparing this table, we assumed that each well could operate for 20 hours a day. Since the yield of each weli had been measured several times during the groundwater studies, we could calculate the total monthly abstraction capacity per polygon.

The fifth column of Table 54 shows the net quantities of surface water (iSRF) to be supplied to the polygons.These quantities also represent the net surface water demand of the polygons. For the critical month of August, the net surface water demand and supply were calculated by taking, respectively, 14.5 and 3.9 per cent (Table 72) of the net iSRF values of Table 54. The difference between the polygonal demand and supply was then determined and compared with the polygonal well capacity. Where this well capacity was not sufficient to meet the difference between supply and demand, it was easy to see what additional capacity was needed. We could then divide that additional capacity by the average yield of the wells in each polygon (Table 73) to find the number of extra wells that would be required. The results of our calculations are shown in Table 74.
table 73. Maximum monthly yield of existing deep wells per poiygon

| Polygon No. | Number of wells | Total yield ( $1 / \mathrm{sec}$ ) | Monthly yield (m $\mathrm{m}^{3}$ month) | Average well yield $\text { (m } \mathrm{m}^{3} / \text { month }$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 126.5 | 273,240 | 45,540 |
| 2 | 18 | 868.0 | 1,874,880 | 104, 160 |
| 3 | 16 | 828.0 | 1,788,480 | 111,780 |
| 4 | 7 | 437.0 | 943,920 | 134,846 |
| 5 | 14 | 865.0 | 1,868,400 | 133,457 |
| 6 | 14 | 609.5 | 1,316,520 | 94,037 |
| 7 | 7 | 362.0 | 781,920 | 111,703 |
| 8 | 3 | 87.0 | 187,920 | 62,640 |
| 9 | 4 | 271.0 | 585,360 | 146,340. |
| 10 | 8 | 566.0 | 1,222,560 | 152,820 |
| 11 | 12 | 662.5 | 1,431,000 | 119,250 |
| 12 | 21 | 849.0 | 1,833,840 | 87,326 |
| 13 | 9 | 315.0 | 680,400 | 75,600 |
| 14 | 12 | 389.0 | 840,240* | 70,020 |
| 15 | 11 | 351.0 | 758,160 | 68,924 |
| 16 | 4 | 164.0. | 354,240 | 88,560 |
| 17 | 4. | 203.0 | 438,480 | 109,620 |
| 18 | 6 | 224.0 | 483,840 | 80,640 |
| 19 | 7 | 540.0 | 1,166,400 | 166,629 |
| 20 | 10 | 675.5 | 1,459,080 | 145,908 |
| 21 | 6 | 334.0 | 721,440 | 120,240 |
| 22 | 11 | 467.0 | 1,008,720 | 91.702 |
| 23 | 6 | 247.0 | 533,520 | 88,920 |
| 24 | 0 | 0 | 0 | 0 |
| 25 | 4 | 230.0 | 496,800 | 124,200 |
| 26 | 9 | 427.0 | 922,320 | 102,480 |
| 27 | 0 | 0 | 0 | 0 |
| total |  |  | 23,971,680 |  |

NOTE: It is assumed that the welle can operate 20 hours/axy $=$ 72,000 sec/day.

TABLE 74. Reguired additional groundwater abstraction capacities to cover maximum monthly water deficiencies in a dry year with MXRIV $=150^{\circ} \times 10^{6} \mathrm{~m}^{3}$ and $\mathrm{MXD}=\mathrm{RHS} 1$


As can be seen from Table 74 , additional groundwater recovery would be required in all the polygons in question. The total extra quantity is 4.751 million m ${ }^{3}$ a month, for which 47 new wells would need to be sunk.

Similar calculations were made for the same dry year, but now with the water demand defined by MXD $=$ RHS 2 (see Table 67). That table' reveals that polygons $1,2,4$ to 11,14 , and 15 belong to the category that would suffer from the shortage of surface water. The results of these calculations are shown in Table 75.

| Polygon supplied by river or river \& wells | Water <br> demand in Auguse (14.5\% OF uet iSRF) | Water supply in August (3.9Z of net iSRF) | Deficiency | Capacity of existing wells | Required additional capseity | Required number of wella |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - . | . | $\pi$ i | 11 i 0 | n m ${ }^{3}$ |  |  |
| 1 | 0.589 | 0.158 | 0.431 | 0.273 | 0.158 | 4 |
| 2 | 2.835 | 0.763 | 2.072 | 1,875 | 0.197 | 2 |
| 4 | 0.581 | 0.156 | 0.425 | 0.944 | 0 | 0 |
| 5 | 2.112 | 0.568 | 1, 544 | 1.868 | 0 | 0 |
| 6 | 2.307 | 0.620 | 1.687 | 1.367 | 0.370 | 4 |
| 7 | 2.231 | 0.600 | 1.631 | 0.782 | 0.849 | g |
| 8 | 1.185 | 0.319 | 0.866 | 0.188 | 0.678 | 11 |
| 9 | 0.810 | 0.218 | 0.592 | 0.585 | 0.007 | -1 |
| 10 | 1.518 | 0.408 | 1.110 | -1.223 | 0 | 0 |
| 11 | 1.830 | 0.492 | 1.338 | 1.431 | 0 | 0 |
| 14 | 0.075 | 0.020 | 0.055 | 0.840 | 0 | 0 |
| 15 | 1.704 | 0.458 | 1.246 | 0.758 | 0.488 | 8 |
| TOTAL | . | . | . |  | 2.747 | 38 |

The total extra g'roundwater abstraction capacity requited is $2.747^{\prime}$ million $\mathrm{m}^{3}$ a month, but it is only needed in the polygons $1,2,6$ to 9 , and 15 , the others having sufficient existing well capacities. To obtain this extra capacity, 38 new wells - or 9 less than with RHS 1 - would be needed.

It is obvious that this proposed groundwater abstraction in polygons that are to receive surface water will affect both the water table in those areas and the total net benefit of the solutions. These aspects, however, were not further analysed in our study.

## 12. Discussion

### 12.1 Strong and weak points of the applied techniques

Computer simulation, like its alternatives of analytical models, experimentation, or reliance on experience and intuition, has its strong and weak points and its limitations. Some of these limitations depend on the type of problem under study; others are of a more general kind, the lack or poor quality of basic data being the most common.

The single greatest criticism of simulation is that concerning the validity of the model. Anyone familiar with the subject knows that the critics of modelling are quick to point out that the data on which the models were developed were too poor and partially even lacking or that too many assumptions were made, thus making the results unceliable and erroneous. But rejecting this technique and resorting to other methods does not mean that the planning engineer or his customer will be much better off: the data remain the same and so do the validity problems. It cannot be denied that in literature the weaknesses of models are often hidden in opaque sentences or are not mentioned at all, that oversimplifications are made and not explained, or reporting is too brief for the outsider to be able to check the input data and the assumptions made.

The models described in this publication have their limitations and weaknesses too: simplifications and assumptions had to be made, but it will not be difficult for the reader to discover them.

In our groundwater model, for instance, we assumed an unconfined aquifer throughout the basin, neglecting any vertical flow component. This is an oversimplification of the real conditions because in some parts of the plain, more specifically in the middle and the south, the deep aquifers are confined or semiconfined, as is demonstrated by a number of wells which are artesian or even free-flowing. The assumed two-dimensional flow poses a validity problem for those areas: the water table elevations generated there may be subject to error. It was not possible, however, to develop a model that could take these multiple aquifer systems into account because neither their precise lateral extent nor their physical characteristics were known.

Another weakness of the groundwater model is that little precise information was available on surface water distribution, consumptive use by the crops, cultivated area,water losses, and water percolation. No accurate historical monthly net deep percolation values could be calculated;instead, average annual percolations were estimated and the computer interpolated the required intermediate values.

There were only four years of monthly water table records available to calibrate the model, and in these data were many gaps. Their values then had to be obtained by correlation with water table records taken from other wells.

Because of this lack of basic data, the network developed for the model was limited to only 27 control points. The water table elevations generated for these points are supposed to be representative of axeas varying from 2,000 ha to 15,000 ha, which is not realistic.

The groundwater model is therefore one from which no highly precise answers can be expected. It was, however, the best that could be made under the circumstances and was certainly adequate for the purpose of our study.

The reader will notice similar shortcomings in the linear programing model.Only 22 years of river flow records from a gauging station far upstream in the valley were available. Downstream of this station,tributaries join the main river; water losses occur through evaporation and percolation; flow measurements in the numerous braiding channels of the river mouth could not and cannot be taken; so the real quantities of.surface water entering the Plain are not known. The 22-year set of data that were used in the model were found by correlating the measurements from the gauging station with a number of measurements taken in the downstream tract, upstream of the braiding channels.

Irrigation studies require monthly or fortnightly data on river flows, not the annual figures we used in the model. We admit that this is a serious shortcoming and is one we recognized from the beginning. Working with annual flows means that one cannot be certain whether the flow on a particular date will be able to meet the demands of agriculture. We have shown that surface water deficiencies occur from June to November and that these shortages can be overcome by sinking a number of additional wells and using groundwater instead of surface water in that period.

The models could have been developed on a monthly or fortnightly basis but this would have required many more data than we had at our disposal. It would also
have complicated the undertaking considerably. So, as this was our first attempt to link two entirely different models, we decided for the sake of convenience and simplicity to work with annual quantities.

The $m^{3}$ prices of water are based on 1966 data and are therefore obsolete; they can, however, easily be updated and the study repeated with the new prices. A more serious shortcoming is that the $\mathrm{m}^{3}$ prices of surface water are based on a new canal network that is to cover most of the Plain. The water supply solutions obtained indicate that only part of this network will be needed, which means that the prices of this water are too high.

Discrepancies also occur between the actual and used $\mathrm{m}^{3}$ prices of well water, especially in those areas where substantial changes in water table were predicted. Steeply rising or falling water tables will decrease or increase the $\mathrm{m}^{3}$ price of this water.

For the purpose of actual planning, the $m^{3}$ prices of surface water and groundwater should be recalculated on the basis of the actually required canal network and the actual depth to water table. With these new prices, another computer run can be made to obtain the water supply solution, which can then be tested for its technical feasibility.

The cost of artificially recharging the groundwater basin was not included in the study, nor were the financial implications of resetting farmers.

Finally, the reader may disagree with certain of the assumptions we made: the irrigation efficiencies, for instance. But this and similar criticisms do not pose any special problems. It is one of the strong points of the models that alterations in water quantities can easily be made and new computer runs performed; all that is needed is to recalculate a specific set of coefficients.

Despite the many admitted shortcomings of our study, its strongest point lies in the fact that by linking the two models we could immediately test any water supply scheme generated by the linear programning model on the groundwater model to find out the effect it would have on the water table.

### 12.2 Comparison of the feasible solutions obtained

In Chapter 10 we showed that with an annual surface water availability of $220 \times 10^{6} \mathrm{~m}^{3}$ and an optimal use of the groundwater basin, two feasible water
supply solutions exist. Both allow some 26,000 ha of good quality land to be irrigated and kept under irrigation even in dry years. The most crucial decision to be made in planning the agricultural development of the Plain is the choice between these two solutions, because they imply two alternative policies of distributing the available water.

Defined in the model as RHS 1 and RHS 2,these policies mean either supplying the water to the best land regardless of the number of farmers living on that land, or supplying the water to the farmers living on the best lands, each farmer possessing 3.85 ha. Table 76 compares the results of the solutions.

TABLE 76. Comparison of the feasible water supply solutions obtained

|  | RHS 1 | RHS 2 |
| :---: | :---: | :---: |
| Maximam volume of surface water available | $220 \times 10^{6} \mathrm{ma}^{3}$ | $220 \times 10^{6} \mathrm{~m}^{3}$ |
| Total area of Class I and Class II land | 57,487 ha (100\%) | 57,487 ha (1007) |
| Total irrigated area | 25,842 ba (45\%) | 26,066 ha (455) |
| Total non-irrigated area | 31,645 ha (55\%) | 31,421 ha (55\%) |
| Total volume of water supplied | $286.5 \times 10^{6} \mathrm{~m}^{3}$ | $286.2 \times 10^{6} \mathrm{~min}^{3}$ |
| Value of objective function (net benefit) | 606,330,900 R1s | 536,001,300 R1s |

The main difference between the two solutions is the much higher net benefit of RHS 1. As can be seen from Table 38 , its potential for agricultural production per polygon and therefore its polygonal water demand is higher than for RHS 2 . In allocating the water for the RHS 1 scheme, the computer selected first the polygons with the highest shadow prices of water (=net imputed value of an additional quantity of water) and then, in a descending order of rank and as long as water was available, the other polygons. The available water quantities for RHS I and RHS 2 being the same, the computer allocated the extra water needed per polygon in RHS $\mid$ at the expense of the polygons with the lowest shadow prices of water in an ascending order of rank. The polygons with the lowest shadow prices, which received water in the RHS 2 solution, did not in the RHS 1 solution. The RHS 1 solution, in fact, means a high concentration of agricultural production in the polygons with the highest shadow prices, which explains its economic superiority.

The principal and ultimate aim of any development project is undoubtedly the welfare of the people concerned. It is not merely a matter of evaluating the purely economic implications; of at least equal importance are the social and human aspects of reallocating land and resettling farmers.

The calculations for the RHS 1 scheme indicate that of the 7,986 famers in the Plain, 5,742 would receive water and 2,244 would not. Besides creating a severe employment problem, this scheme would also, mean a considerable variation in farm size: from 1.42 ha to. 13.55 ha.

If we assume an equitable distribution of land, i.e. a farm size of 3.85 ha as was imposed in alternative RHS 2 , then 7,005 farmers could be supplied, leaving 981 farmers unsupplied. However, of the 7,005 farmers only 4,774 live in the supplied polygons and 3,212 in the unsupplied, peripheral polygons. With farms of 3.85 ha there would be sufficient water to supply another 2,231 farmers, who could be resettled from the unsupplied areas to the middle of the Plain. For the remaining 981 facmers other employment must be found. If the farm size were to be reduced further, to 3.24 ha, these 981 farmers could also be resettled in the middle of the Plaja.

The RHS 2 scheme indicated that of the total number of farmers in the Plain, 6,981 would receive water and 1,005 would not. This number differs only slighty from the 981 found for RHS 1.

Although, because of its higher net benefit, RHS I seems at first sight to be superior to RHS 2, it implies a much more intensive resettlement programue. The cost of this resettlement was not calculated in the study but it will no doubt substantially reduce the net benefit of RHS I. RHS 2 also implies a certain resettlement but this will take place within the supplied areas; there will be no inflow of farmers from unsupplied areas and many if not most of the existing villages can be maintained. Even so, any resettlement will reduce the net benefit of RHS 2 .

It will be recalled, however, that the canal network required for RHS 1 will be shorter than that for RHS 2. (The latter will also be shortened but less than for RHS 1.) Consequently, the surface water supply for RHS 1 will be less costly than for RHS 2, and these lower costs will raise the net benefit of RHS 1.

The obvious adyantage of RrS 1 is that all activities would be concentrated in a single block in the middle of the Plain, thereby reducing not only the length of the canal system but also the area in which infrastructural improvements would
be needed. However, since the financial implications of resettlement, shortening the canal network, and infrastructural improvements were not elaborated, it is not possible to say definitively which is the better scheme.

The various peripheral areas, whose abandonment has so explicitly been proposed by our study, contain many villages whose history can be traced back for thousands of years. Some of these villages are quite prosperous and their populations increase in spite of their relatively high water supply costs. Other villages are in a depressing process of decay; their canals are no longer maintained, most of their fields, farm houses, and community buildings are abendoned, and the young generation is migrating to other places.Farther to the south,ruins of houses and structures can be seen, testifying to previous attempts to develop some form of irrigated agriculture in these marginal areas of the Plain.

This socio-economic and human panorama undoubtedly deserves governmental efforts to establish a new, modern, reliable, and efficient water supply system. But with water such a severe limiting factor, no matter what policy is adopted, it will still not eliminate the necessity of finding employment for some 1,000 farmers who cannot be supplied with water. Of course, one might think of trying to save water by increasing the irrigation efficiency, or supplying less water than the maximum demand, or even introducing cropping patterns that require less water. Such measures, however, are not realistic and one has to reckon with the problem of resettling farmers from the unsupplied and marginal areas to towns or other places where employment must be found for them.

The proposed farms of 3.85 ha are rather small and we have foreseen that in a second development stage they might be enlarged to 6 ha. The water demands of such a policy were calculated and indicated as RHS 3, but since the RHS 1 and RHS 2 policies already create the problem of forcing 1,000 farmers to find other work, it is clear that RHS 3 would only aggravate matters. Besides, RHS 3 can only be considered for implementation if more surface water can be made available to the Plain.

### 12.3 The models as tools for further planning

The water supply schemes discussed in this publication are not final solutions in themselves, but they can form an excellent basis for further planning. They point out the direction in which the ultimate solution of optimal joint development and distribution of surface water and groundwater must be sought.

They have made clear, for instance, the economic benefits that can be obtained by concentrating development activities in a compact block on the best land in the middle of the Plain. They have also revealed that if any meaningful development is to be achieved, some 1,000 farmers will have to find work outside agriculture. This emphasizes the complexity of the historical transition from an' ancient and outdated water supply system of qanats, earthern canals, and severe water losses to..a new, integrated, and efficient system of lined canals and deep wells.

The potential resettlement candidates must be carefully assessed by the planning . agent in a village to village approach. The existing social, human, and community patterns will have to be evaluated in situ, to reach a maximum consent with the families and communities involved.It might be opportune in this respect to compromise in cases where, say, an extra 4 km of canal and somewhat higher water transportation costs may save a particular village from being uprooted and: relocated.

The resettlement activities must be carefully planned. Their many legal, political, economic, and human aspects, which could not be included in our model, will have a decisive impact on the size and shape of the water supply system and hence on the water supply solution that is finally accepted. Resettlement costs could, in principle,be considered an alternative to coṣly water transportation investments. Resettlement costs will be decided by such considerations as the size and quality of the farmers' new housing and the standard of communal services. The costs will also depend on the planning agent's ability to find non-agricultural employment for displaced farmers. In a newly developing area, there will be a need for organizations for the supply of inputs, for the marketing, storage, and processing of agricultural produce, for land preparation and the hiring of farm machinery, and for social services. All such organizations will offer employment opportunities and may, in fact, absorb so many former farmers that the number needing resettlement is less than wis originally expected. The final number of farmers to be resettled and the final design of the water supply system will ultimately determine the farm sizes and target incomes.

Planning the development of agriculture in the Plain is not just matter of the design, implementation, and operation of a modern water supply system. It is much more. It is the transition from a complex and traditional way of farming to new, modern agriculture, and its social and psychological aspects must be Eully understood before decisions on costly investments are taken.

We believe that, in this whole intricate process of planning, the modelling techniques discussed in this publication have a valuable role to play. They can provide a sound footing on which decision-makers can base their decisions. The linear programing model possesses great versatility in the consideration of alternatives. In the light of new information that may become available, for instance, the river flows can be re-analysed and changed if need be. Similarly, cost coefficients can be changed: the costs of replenishing the groundwater basin can be calculated and included in the $m^{3}$ price of well water; the costs of surface water can be recalculated for the appropriate canal system. With these new values, the "optimum" solution can be redetermined. Percolation coefficients can also be changed, other farm sizes chosen, or other cropping patterns applied. Chese will result in a new set of water-demand values. The resettlement problem and other legal and institutional constraints can be quantified and evaluated by solving the problems with and without the constraints. If, for some reason, a peripheral village cannot be abandoned and its farmers resettled elsewhere, it is easy to force the computer to meet the water demand of that village.

It will have become clear by now that planning a water supply system for the development of agriculture in an arid region is by no means a simple matter. The many hydrological, economic, agronomic, sociological, and engineering aspects involved in such an undertaking must all be carefully weighed. The desire to improve the lot of poor farmers through the control of water and the application of modern technology often leads to disaster - hydrologic or human. The combination of technique and good intentions does not guarantee success.

## Symbols

| SYMBOL | DESCRIPTION | DIMENSION |
| :---: | :---: | :---: |
| A | area | $L^{2}$ |
| a | soil constant | $\mathrm{L}^{2.5} \mathrm{~T}^{-1}$ |
| a | linear programming coefficient | dimensionless |
| B | number | dimensionless |
| b | linear programming coefficient | dimensionless |
| C | constant | dimensionless |
| c | cost of activity, linear programming coefficient | dimensionless |
| D | thickness of water bearing layer | L |
| d | average grain size | L |
| e | linear programming coefficient | dimensionless |
| g | acceleration due to gravity | $\mathrm{LT}^{-2}$ |
| h | hydraulic head | L |
| i | number | dimensionless |
| j | number | dimensionless |
| K | hydraulic conductivity | L $\mathrm{T}^{-1}$ |
| k | specific permeability | L $\mathrm{T}^{-1}$ |
| $\mathrm{k}_{0}$ | capillary conductivity at $\psi=0$ | L $\mathrm{T}^{-1}$ |
| L | length | L |
| Q | discharge | $\mathrm{L}^{3} \mathrm{~T}^{-1}$ |
| S | storage coefficient of aquifer | dimensionless |
| T | $=\mathrm{KD}=$ transmissivity of aquifer | $\mathrm{L}^{2} \mathrm{~T}^{-1}$ |
| t | time | T |
| V | velocity of flow | L $\mathrm{T}^{-1}$ |
| $v$ | velocity of flow in the soil | L $\mathrm{T}^{-1}$ |
| W | width |  |
| X | variable | dimensionless |
| $\mathrm{x}, \mathrm{y}, \mathrm{z}$ | cartesian coordinates ${ }^{\text {c }}$ |  |
| Y | conductance | $\mathrm{L}^{2} \mathrm{~T}^{-1}$. |
| $z$ | depth to the water table | L |
| $a$ | soil constant | $L^{-1}$ |
| 7 | dynamic viscosity of water | M L |
| $\theta$ | water content of the soil (volume \%) | dimensionless |


| SYMBOL | DESCRIPTION | DIMENSION |
| :--- | :--- | :--- |
| $\rho$ | mass density of water | $\mathrm{ML}^{-3}$ |
| $\psi$ | suction of soil moisture | L |
| $\psi_{a}$ | suction at the air entry point | L |

Symbote used in the comprehensive linear programming model

| i PRD | agricultural production in polygon i |
| :--- | :--- |
| i WEL | well water supply in polygon $i$ |
| i SRF | surface water supply in polygon $i$ |
| i DEM | water demand to produce crops in polygon $i$ |
| i MXD | maximum water demand of polygon $i$ |
| i MXSFY | maximum "safe yield" of polygon $i$ |
| MXRIV | maximum quantity of river water available |
| MXSFY | maximum "safe yield" of groundwater basin (=difference <br> between total inflow and outflow of groundwater) |
| j SRF | surface water supply to j polygons |
| j WEL | well water supply to $j$ polygons |

RHS I water supply to the best quality land (Class I and Class II), regardless the number of farmers living on these lands

RHS 2 water supply to all the farmers living on the best quality land (Class I and Class II), each farmer possessing 3.85 ha, regardless of where these lands happen to be situated in the Plain

RHS 3 the same as RHS 2, except that the farm size is 6.0 ha
a fraction of the groundwater that is available at the field in polygon $j$ (or i), if a unit volume of groundwater in polygon $j$ (or i) is recovered
$b_{j}$
sum of the portions of river water that percolate to the water table in polygon i, if a unit volume of river water is released from the weir to supply polygon $\mathbf{j}$
$e_{i j} \quad t$ he portion of the river water that percolates to the water table in polygon i, if a unit volume of river water is released from the weir to supply polygon $j$
$c_{i} \quad$ the portion of the river water that is available at the field in polygon i, if a unit volume is released from the weir

## Authors index



## Subject index



Artificial recharge $88,89,174,219$
Asymmetric grid
network $\quad 36,44,45,48,49$
Bakhtiary formation 7, 44
Canal system 107, 108
Capillary rise 55 - 57
Classification
of groundwater 27, 28
of land 12
Climate $\quad 13,53$
Comprehensive water supply model97

Constraints 67, 89
Continuity equation 35,36
Conveyance losses 127-136
Cost
of agricultural production 98,106
of surface water 99,107-117
of water
supply $152,157,167,170,200,204$
of well water $\quad 99,118-124$
Cropping patterns
average net income per crop 85,86
net income per crop $84,88,{ }^{\prime} 98$
subregional
82-84

[^4]Dam storage . 19
Darcy's equation 34
Damavand Rud 4, 5, 15, 19
$\begin{aligned} & \text { Discharge } \\ & \text { of groundwater }\end{aligned} \quad 33,59,62$
of Jaj Rud 15,16
of qanats 22-26,54
of wells 24,25
Drainage base 63
Evaporation $\quad 13,14,55,57$
Expansion of irrigated area 224
Farm (model) 66
surface water
availability at 135
Finite differences method 36,37
Flow equation 35,36
Elow velocity 34,35
Fortran programme 39,42
Gauss-Seidel method 40
Groundwater
availability $100,125,141,142$
balance . 53,59,60
classification 27,28
inflow 33,59,61,62
in storage 21
mining $\quad 142,180,183,212$
model . 31,44
outflow 33,59,62
quality 26
recharge ' 54
recovery $21,24,25,89,93,231$
system . 33
table 52
Hezardareh formation , 7
Hydraulic head 35
Impervious base 62, 63
Importation of water 224
Irrigation water
shadow prices of $\quad 74$
Jaj Rud $\quad 4,13,19$
dam 13,19
deposits 7
discharge $\quad 15,16$
Gumbel distribution 18
log normal distribution 17
losses to desert 20,23
percolation losses
19, 59
29, 30
water rights 13

Land classification
Land use
selecting a policy $142,161,162$
Lar Rud
5, 20
Linear programing see also Comprehensive water supply model, Cost, cropping patterms, Matrix, Model, Shadow prices, Test model

Linear programming
66, 87, 97
activities $67,68,88,98,103$
coefficients 99-105,127,134-136
constraints $67,89,99,100,103,126$
parametric $69,75,76,139,140,224$
objective function
67,98
Matrix
of comprehensive model $103,104,137$
of test model
69,92
Maximization 98

Maximum water demand polygon

125,126
Maximum "safe
yield"
of basin $\quad 100,103,148,150,189$
of polygon $102,103,148-150,189$
Model
calibration 63
farm . 66
groundwater basin 31, 38
simulation
31, 33
Net benefit
200, 201
Net deep pereolation see also Percolation

Net deep percolation output for caculating . 149

Net income 74
per $\mathrm{m}^{3}$ of water $84,85,86,88,98$
Objective function 67,98
Optimal production plan 72
Optimal water supply.solution of comprehensive model 146,147,189-191

$$
\text { of test model } 93
$$

of three farm types ..... 73
testing for
feasíbility
148,167,192

Overall groundwater balance
$53,59,60$
Paranetric linear
programming $69,75,76,140,224$
Percolation 35, 132
downstream of farm
group inlet.
131-136
in canals 53, 54, 127-131, 135
in fields $\quad 53,54,131-135$
in polygons $60,100,132-135,150,189$
in river bed 19
net deep $37,43,60,62,100,130,132$,
$135,148-150,189$
Polygon
network 36,45,89
number 46
size 46
Precipitation 14
Qanat
construction . 22
definition • 21
discharge 22-26, 54
Quaternary deposits 7
Rainfall . 14
recharge from 53
Relaxation coefficient 39,40
Residual term 39
Resources
availability $100,125,126,135$, 139-143

River discharge $\quad 15,16,59$
River water
quantities
71
Sensitivity tests 69;78
Shadow prices $72,74,158,159,206,207$, 229

Simulation 31
Soils 11
Specific yield of waterbearing layers . 50

Storage
change in 37, 59
coefficient $35,37,50-52$
Surface water
availability $20,100,125,139,140$
deficiency
$12,13,186$

| Systems approach |  |
| :---: | :---: |
| Test model | 87, 90 |
| Tolerance level | 39, 40 |
| Total net income | e 74, 77, 98 |
| Transmissivity of waterbearing - |  |
| Velocity of flow | w 34, 35 |
| Water, importatio | ion of 224 |
| Water consumption tests | on coefficients $78$ |
| Water demand alternatives fo of crop pattern of crops per polygon | $\begin{array}{rr}\text { for a model } \begin{array}{r}\text { farm } \\ \text { rns }\end{array} & 70 \\ & 85, \\ 86 \\ 86,95,98,125,126,186\end{array}$ |
| Water losses |  |
| in conveyance |  |
| systern | 53,54,127,129-131 |
| in fields | 53, 54 |
| in polygon | 60,100,132,150 |
| in river bed | 19 |

Water rights
Water supply see also Optimal
water supply solution

## Water supply

alternative schemes
of $70,71,101,144$
alternatives for a
model farm
70

Water supply solution
adjustment 152-153,200,208
Water table
data 52-53
depth 57,58
gradient $\quad 46,47$
Watershed of Jaj Rud 13
Well
annual discharge $\quad 24,25,54$
capacity per polygon 231
observatión 48,52
pumping tests on 48
water prices . . 71

## References

ALLENBACH, P. (1966): Geologie und Fetrographie des Damavand und seiner Umgebung. Mitt.Geol.Inst.der Eidgen.Techn. Hochschule und der Univ.Zürich.

AN-MIN-CHUNG (1963): Linear Programing. Ch.E.Merrill Books Inc., Columbus, Ohio.

ANONYMOUS (1964): Etude hydrogéologique par prospection êléctrique de la région. de Varamin. Comp.Gënẹrale de Gẹophysique. Paris. 29 pp .

ANONYMOUS (1967): Nater legislation in Asia and The Far East. United Nations. Water Resources Series No.31. Part I.

BEAUMONT, P. (1968): Qanats on the Varamin Plain, Iran. Trans.Inst. British Geographers. 45:169-179.
(1971): Qanat systems in Iran. Bull. of the Int.Assoc. of Sci. Hydrology 16:39-50.
(1973): A traditional method of groundwater utilization in the Middle East. Groundwater 11, 5:23-30.

CHUN, R.Y.D., E.M.WEBER, and K.MIDO (1963): Computer tools for sound management of groundwater basins. Int.Assoc. of Sci.Hydr.Berkeley. Publ. No.64:427-437. DAVIS, S.N., and R.J.M.de WIEST (1966): Hydrogeology, John Wiley \& Sons Inc., New York.

DELLENBACH, H. (1963): Contribution à l'êtude gêologique de la rêgion situêe à 1 'est du Teheran (Iran). Thesis. Univ.Strassbourg.

DE RIDDER, N.A. (1971): Groundwater Resources. Final Report. Food \& Agric. Org. of the U.N., Rome. 219 pp .
(1968): Simulation of the Varamin Ground Water basin, Iran, on a digital computer. Food \& Agric.Org. of the U.N., Rome. $55 \mathrm{\rho p}$.
A. EREZ, R.Y.D. CHUN, and E.M. WEBER (1969): A computer approach to the planning for optimum irrigation water supply, development, and use in the Varamin Plain. Iran. Mimeograph. Food \& Agric.Org, of the U.N., Rome, 28 pp.

DE WIEST, R.J.M. (I965): Geohydrology. John Wiley \& Sons, Inc. New York.
DOMENICO, P.A. (1972): Concepts and models in groundwater hydrology.
McGraw-Hi11 Co., New York. 405 pp.
EMADT, A.S. (1966): Groundwater investigations of the Varamin Plain. Final Report. Min. of Water \& Pòwer Gen. Adm. of Water Resources. Hydrogeol.Dept., Teheran. 183 pp .

EREZ, A. (1967): Farm management studies. Part I: Varamin Survey Analysis. Mimeograph. Food \& Agric.Org. of the U.N., Rome.
—... (1967): Farm management studies. Part IV: Water supply test model using linear programming techniques. Mimeograph. Food \& Agric.Org. of the U.N., Rome, 18 pp.
; and M. BAHADORI (1967): Farm management studies. Part VII: Results of the socio-economic survey of the Varamin area. Mimeograph. Food \& Agric. Org. of the U.N., Rome:
—— (1968) : Farm management studies. Part X: Varamin's water supply planning. Mimeograph. Food \& Agric. Org. of the U.N., Rome. 64 pp.
(1969): Farm management studies. Part XI: Computation progranme for Varamin water supply planning. Mimeograph. Food \& Agric.Org. of the U.N., Rome', 20 pp.

FOWLER, L.C., and V.E.VALANTINE (1963): The coordinated use of groundwater basins and surface water delivery facilities. Int. Assoc. of Sci.Hydrology. Berkeley, Publ.No.64, 376-383.

GARVIN, W.W. (1960): Introduction to linear programming, McGraw Hill, New. York.
GASS, S.I. (1958): Linear programming. Methods and appifcations. McGraw Hill, New York.

HALL, W.A., and J.A.DRACUP (1970): Water resources systems engineering. McGraw Hill, New York.

HUFSCHMLDT, M.M. and M.B.FIERING (1966): Simulation techniques for design of water resources systems. Harvard Univ. Press, Cambridge, Mass.

LLOYD, J.W., D.S.H.DRENNAN, and B.M.U.BENNEL (1966): A groundwater recharge study in NE Jordan. Proc. Inst. Civil Engrs. 35:6:5-631.

MACNEAL, R.H. (1953): An asymetrical finite difference network. Quarterly J. of App1. Mathem. XI, 3: 295:310.

NIOC (1959): Geological Map of $\operatorname{Ir}$ an $1: 2,500,000$ with explanatory notes. REMSON, I., G.M.HORNBERGER, and F.J.MOLZ (1971): Numerical methods in subsurface hydrology, Wiley-Interscience, New York.

RICHARDS, L.A. (1954): Saline and alkali soils. Agric.Handbook No. 60. Denver, Col.

RICHARDSON, L.F. (1910): The approximate arithmetical solution by finite differences of physical problems involving differential equations with an application to the stresses in a masonry dam. Phil. Trans, Royal Soc.Ario. p. 307-357.

RIEBEN, E.H. (1966): Geological observations on alluvial deposits in Northern Iran. Report No.9, Geol.Survey of Iran. Teheran, 39 pp.

RIJTEMA, P.E. (1965): An analysis of actual evapotranspiration. Thesis. Univ, of Agric., Wageningen.

SCHULZE, F.E., and N.A.de RIDDER (1974): The rising water table in the West Nubarya area of Egypt. Nature and Resources $X, 1: 12-18$.

SPIVEY, W.A. (1963): Linear programming. An introduction. MacMillan Comp., New York.

THOMAS, R.G. (1973): Groundwater models. Irrigation and Drainage Paper 21. Food \& Agtic.Org, of the U.N., Rome. 192 pp.

TODD, D.K. (1959): Groundwater hydrology, John Wiley \& Sons, New York.
TYSON, H.N., and E.M.WEBER (1964): Groundwater management for the Nation's future. Computer simulation of groundwater basins. J.Hydraulics Div.Am. Soc. Civil Engrs. 90 (HY-4), 59-78.

VALANTINE, V:E. (1964): Groundwater for the Nation's future. Effecting optimum groundwater basin management. J.Hydraulics Div.Ar. Soc.Civil Engrs. 90 (HY-4), 97-105.

VAÑ VUGT (1969): Agronomy of the Varamin Plain, Iran. Final report. Food \& Agric.Org. of the U.N., Rome. 80 pp.

VOJSIC, M.S. (1969): Irrigation data on the Varamin Plain. Mimeograph. Food \& Agric.Org, of the U.N., Rome. 32 pp .

VOJSIC, M.S. (1969): Irrigation research at Deh Vin, Fall 1967-1968. Report. Food \& Agric.Org. of the U.N., Rome. 22 pp.

VUKCEVIC, M. (1969) : The Jaj Rud watershed. Report. Food \& Agric.Org. of the U.N., Rome. 53 pp.
-... (1970): Surface water resources of the Varamin Plain. Final Report. Food \& Agric.Org. of the U.N., Rome. Vol.II, Part I. 99 pp. WEBER, E.M., H.J.PETERS, and M.L.FRANKEL (1968): California's digital computer approach to groundwater basin management studies. Symposium on use of analog and digital computers in hydrology. Tucson, Arizona. Dec.8-15. Int.Assoc. of Scientific Hydrology, Gent/Brugge, Belgium. Publ. No.80:215-223.

WIND, G.P. (1955): A field experiment concerning capillary rise of moisture in a heavy clay soil. Neth.J.of Agric.Sci, 3.1.


[^0]:    The month indicate planting time
    Feed units

[^1]:    in the LP solution stomer gratits
    mainity teaf vegetables ("sabsi" in persian)

[^2]:    1 iines 3.85 ha per farm. ( 3.2 ha in Polygon 1, 3.5 ha in Polygort 10-14)
    2 itimes 6.00 ha per farm (3.2 ha in Polygion 1, 5.4, 5.4, 4.1, 4.7; 5.2, and 3.8 in respectivety potyens $10-14$, and 221

[^3]:    a.s.0

[^4]:    Crops
    consumptive use of $60,99,100,125$
    water requirement of $85,86,107$
    Cycling river flows $171,186,215$

